AN INVESTIGATION OF THE RELATIVE EFFECTIVENESS OF THREE METHODS OF UTILIZING LABORATORY ACTIVITIES IN SELECTED TOPICS OF JUNIOR COLLEGE MATHEMATICS

By

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A DISSERTATION PRESENTED TO THE GRADUATE COUNCIL OF THE UNIVERSITY OF FLORIDA IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

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To

Mother, Daddy, and Bruce

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Joan Marie Golliday

August, 1974

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Cochairman: Dr. Elroy J. Bolduc
Major Department: Curriculum and Instruction

Purpose of the Study

The purpose of the study was to investigate the relative effectiveness of the mathematics laboratory when used in three different ways in conjunction with a traditional lecture-discussion approach to teach community college freshmen enrolled in a required mathematics course.

Procedures

The sample population for the study consisted of seven sections of a required mathematics course at Santa Fe Junior College, Gainesville, Florida. On the basis of their scores on the Myers-Briggs Type Indicator the subjects were classified as sensing or intuitive. They were also classified as high, average, or low achievers on the basis of their college grade point average. One group, known as the

exploratory-discovery group, received sixty minutes of laboratory experiences followed by thirty minutes of discussion. The second group, known as the verification-application group, received thirty minutes of lecture followed by sixty minutes of laboratory. The third group, known as the combination group, received thirty minutes of laboratory experience both before and after the thirty minutes of lecture-discussion. All groups studied the topic of ratio and similarity for two and one half weeks. They were given both a pretest and a posttest. The resulting mean error scores of the 94 subjects were compared in a 3x3x2 factorial design using analysis of covariance. The pretest error scores were used as a covariate. Scheffé's Method was used to determine the significance of the reductions in mean error score for the various subcategories.

Conclusions

In comparisons of sensing and intuitive subjects without regard to achievement level or method, the sensing subjects did significantly better than the intuitive subjects. Comparisons for all other main effects and interactions were nonsignificant.

The investigation of reductions in mean error scores indicated that the exploratory-discovery group attained a significantly greater reduction than the other two groups. Within the categories of average-achievers and sensing students, the exploratory-discovery group also achieved a

significantly greater reduction than the other two groups. Low-achievers in the combination group achieved a significantly greater reduction in mean error score than those in the exploratory-discovery group. Finally, sensing high-achievers in the combination group achieved a significantly greater reduction in mean error score than those in the verification-application group.

CHAPTER I

INTRODUCTION

General Background of the Problem

In recent years, the general concern over the educative process has produced a number of innovative teaching techniques. Some of these innovations are genuinely new while others are old techniques which have merely been renovated, energized, and generally modernized. Regardless of which technique one chooses to investigate, it soon becomes apparent that all have suffered from the same malady --- namely, little or no experimental documentation of their worth and validity.

At the present time the concept of a mathematics laboratory is again appearing upon the educational horizon. This particular teaching technique has had periods of popularity at least twice within the last century. The initial appearance of this teaching approach is generally associated with the English mathematician John Perry. Perry first promulgated his revolutionary ideas in 1901 in a report on the "Teaching of Mathematics" which he presented to the British Association for the Advancement of Science. His main concern was that too much emphasis was being placed on the

theoretical aspects of mathematics. He proposed that a more meaningful approach would be to teach a combined physics and mathematics course putting the physical or applied aspects first. "Perry favored a laboratory approach, including greater emphasis on experimental geometry, practical mensuration, the use of squared paper to plot statistics, interpolate, discuss slope, and find maximum and minimum values, easy vector algebra, more solid geometry, and the utilitarian parts of geometry" (Mock 1963, p. 131). Apparently, the vast majority of the mathematics teachers in England agreed with Professor Perry and the Perry Movement was soon spreading across England and to America. During the next couple of years, articles dealing with the pros and cons of the laboratory approach abounded. It seemed that the laboratory concept was here to stay. But Perry and his followers had not reckoned with the rigid, unchanging testing system of England's school system. As student scores fell so did support for the Perry Movement. Its popularity lasted less than ten years.

The second emergence of a laboratory-type of instruction came in the early 1940's. This was an era of multi-sensory aids. Topics were taught using movies, film strips, slides, and overhead and opaque projectors. The major shortcoming in this approach was the passive role of the student. All these multi-sensory aids were used by the teachers to demonstrate principles which they expounded rather than as a means of hands-on discovery by the student.

After approximately five years, the novelty of this approach began to subside and the teachers gradually returned to their traditional methods of instruction.

The most recent reemergence of the mathematics laboratory began in the early 1960's. Its spread across the United States has been more gradual than in the past and this in itself may be a healthy sign. The bandwagon approach which has meant disaster in the past has been avoided. Today the mathematics laboratory is viewed as an adjunct to rather than a replacement for the more traditional forms of instruction. Past studies have dealt with the question of whether or not the laboratory method is better than the traditional lecture-discussion technique. The results of most of these studies have been inconclusive. What is needed, in view of today's educational philosophy, are studies to determine how the laboratory may be most effectively used in conjunction with the traditional lecture-discussion method. It was with this in mind that the present study was undertaken.

Statement of the Problem

The purpose of this study is to investigate the relative effectiveness of the mathematics laboratory when used in three different ways in conjunction with a traditional lecture-discussion approach to teach community college freshmen enrolled in a required mathematics course. The laboratory experience will be used as an introduction to a

topic, as a reinforcer, or as both. All subjects are classified by achievement level and personality type, as determined by the Myers-Briggs Type Indicator, so that the interaction of these factors with the various laboratory approaches may be assessed. In particular, as a prelude to the formal null hypotheses found in Chapter III, the following research questions are of interest:

1. Do college freshmen studying ratio and similarity under one sequencing pattern of laboratory experiences and discussion do significantly better than those studying the same topics under different sequencing arrangements?
2. Do high-achieving college freshmen studying ratio and similarity perform significantly better under any one of the three sequencing arrangements?
3. Do average-achieving college freshmen studying ratio and similarity perform significantly better under any one of the three sequencing arrangements?
4. Do low-achieving college freshmen studying ratio and similarity perform significantly better under any one of the three sequencing arrangements?
5. Do college freshmen who have been identified as sensing personality types on the basis of the Myers-Briggs Type Indicator perform significantly better under any one of the three sequencing arrangements when studying ratio and similarity?
6. Do college freshmen who have been identified as intuitive personality types on the basis of the Myers-Briggs

Type Indicator perform significantly better under any one of the three sequencing arrangements when studying ratio and similarity?

7. Do high-achieving college freshmen who have been identified as sensing personality types perform significantly better under any one of the three sequencing arrangements when studying ratio and similarity?
8. Do high-achieving college freshmen who have been identified as intuitive personality types perform significantly better under any one of the three sequencing arrangements when studying ratio and similarity?
9. Do average-achieving college freshmen who have been identified as sensing personality types perform significantly better under any one of the three sequencing arrangements when studying ratio and similarity?
10. Do average-achieving college freshmen who have been identified as intuitive personality types perform significantly better under any one of the three sequencing arrangements when studying ratio and similarity?
11. Do low-achieving college freshmen who have been identified as sensing personality types perform significantly better under any one of the three sequencing arrangements when studying ratio and similarity?
12. Do low-achieving college freshmen who have been identified as intuitive personality types perform significantly better under any one of the three sequencing arrangements when studying ratio and similarity?

In order to test the series of null hypotheses generated by these research questions, 94 college freshmen enrolled in an introductory mathematics course at a Florida community college were identified as sensing high-achievers, sensing average-achievers, sensing low-achievers, intuitive high-achievers, intuitive average-achievers, or intuitive low-achievers on the basis of their college grade point average and the Myers-Briggs Type Indicator. Each student was administered a pretest and a posttest on ratio and similarity. The resulting mean error scores of the eighteen groups were compared in a 3x3x2 factorial design using the methods of multiple linear regression with the pretest scores as a covariate.

Definition of Terms

The following terms will be used throughout the study:

Sensing Subject: a subject who has been classified as a sensing personality on the basis of his Myers-Briggs Type Indicator score.

Intuitive Subject: a subject who has been classified as an intuitive personality on the basis of his Myers-Briggs Type Indicator score.

High-Achiever: a subject whose grade point average at his current community college is greater than or equal to 3.35.

Average-Achiever: a subject whose grade point average at his current community college is between 2.65 and 3.35.

Low-Achiever: a subject whose grade point average at his current community college is less than or equal to 2.65.

Exploratory-Discovery Method: a method of using the

mathematics laboratory as an introduction to a new topic followed by class discussion of what was observed in the laboratory.

Verification-Application Method: a method of using the mathematics laboratory to illustrate and verify topics which have been taught in the classroom.

Combination Method: a method of using the mathematics laboratory both before and after class discussion so that it both introduces and verifies the classroom material.

Mathematics Laboratory: a mode of instruction which uses experiments to aid students in the discovery and/or verification of mathematical concepts.

Need for the Study

An area of concern for teachers of mathematics has been that of helping the student to obtain a better understanding of the mathematics he is studying. It was this concern which produced modern mathematics. With the advent of modern mathematics there were many teaching innovations such as team-teaching, modular scheduling, discovery learning, and the mathematics laboratory. At first these were advocated as replacements for the traditional modes of instruction. But as researchers found, students did not do significantly better, or worse for that matter, under the new methods of instruction. The unfortunate part is that many of these innovations were abandoned because they did not produce better results than the traditional methods. The fact that they were at least as good as the old techniques was completely overlooked.

There have been several studies in which the

mathematics laboratory has been compared to traditional methods of instruction. See Wilkinson (1970), Cohen (1970), Phillips (1970) and Bluman (1971). In all four of these studies there were no significant differences between the two methods of instruction; that is, they were equally effective.

Since the laboratory approach appears to be as good as the traditional lecture-discussion method, it seems reasonable to use both. It was with this premise in mind that this study was conceived. The questions which immediately came to mind were as follows:

1. Is there a best sequence for using both the mathematics laboratory and the lecture-discussion?
2. If there is a best sequencing pattern will it be the same for all achievement levels?
3. Would the best sequencing pattern be related to personality type?

A search of the literature found only two studies which had considered this question of sequencing. See Reuss (1970) and Emslie (1971). Reuss did his work in biology while Emslie did his in physics. There was no experimental research into these questions using mathematics as the vehicle of study.

Since many school systems are committing themselves to the operation of mathematics laboratories, it is essential that the above questions be answered. This study is designed to investigate the role of the mathematics laboratory at the community college level. It is hoped that this

study will stimulate the further research at the elementary, middle, and secondary school levels which is needed.

Organization of the Study

Chapter I has been an introduction to the study, including some general background information, a statement of the problem, definitions, and an explanation of the need for the study. Chapter II is devoted to a review of related research. The results from five studies comparing the inductive method of instruction to the deductive method are reported in the first section of the chapter. In the second section, the results from five studies comparing the mathematics laboratory to traditional methods of instruction are examined, along with two studies that dealt with the sequencing of a laboratory experience with traditional lecture-discussion instruction. The final section is a summary of the first two sections. Chapter III contains the formal null hypotheses, along with a description of the design, the sample population, and the procedures involved in gathering the data. Information about the pretest, the posttest, the experiments used and the Myers-Briggs Type Indicator is presented along with an explanation of the statistical treatment. Chapter IV is devoted to a presentation and analysis of the data. It also includes the results of a questionnaire completed by the subjects in the study. Chapter V contains a brief summary of the study

together with a list of the conclusions reached. Several limitations are cited, and some implications for instruction and future research are discussed.

CHAPTER II

REVIEW OF RELATED RESEARCH

For nearly twenty years, educators and mathematicians have been concerned with the question of whether the traditional deductive method of instruction is better than the inductive approach. There have been studies which found the deductive method to be better, while others found the inductive approach to be significantly better. The vast majority of the research into this area, however, found no significant differences. Most recently, this question has reemerged with reference to the mathematics laboratory. For this reason, this chapter has been divided into three parts. The first section contains a few representative samples of the research done on the inductive-deductive question. The second section deals strictly with research relating to the laboratory approach to instruction, while the final section will be a summary of the results reported herein.

Inductive-Deductive Studies

One of the earliest studies to investigate the relative effectiveness of the inductive and deductive methods of instruction was conducted by Dr. Max Sobel (1956). In this

study, Dr. Sobel investigated the effectiveness of the inductive method of teaching algebra as compared to the traditional deductive method. In order to do this he used fourteen ninth-grade algebra classes in Newark and Patterson, New Jersey. Seven classes were taught by each method and every class except two had a different instructor. The teachers using the inductive approach were given a manual of instruction, an explanation of the study, and numerous illustrative examples to be used. The deductive group used the normal textbook. At the end of four weeks the students were given a test which had been developed by the researcher. A review of I.Q. scores for all students indicated that subgrouping by intelligence level was also possible. An analysis of the data found that bright students learned and retained skills better when taught by the inductive method. For the average intelligence level, there were no significant differences.

In 1965, Krumboltz and Yabroff conducted a study to determine the teaching efficiency of inductive and deductive sequences of instruction with varying frequencies of alternation between problem-solving and rule-stating frames. They also investigated the interaction of these factors with intelligence levels. The experimental sample consisted of 272 students enrolled in an introductory education course at the University of Minnesota. Each student was given the Miller Analogies Test and was categorized as high or low in intelligence on the basis of his score. Four forms of

programmed materials on elementary statistics and test interpretation were randomly distributed to all students. Two forms were inductive with different frequencies of alternation while the other two were deductive with differing frequencies of alternation.

Each student was given a test two weeks after the end of the instructional period. An analysis of the data using analysis of variance found the following results:

1. The high-intelligence group completed their work in significantly less time than the low-intelligence group.
2. The inductive group made significantly more errors than the deductive group.

A similar study to that of Krumboltz and Yabroff was conducted by Koran (1971). Her sample population consisted of 167 students enrolled in an introductory education course at the University of Texas. Each student was given selected measures from the Kit of Reference Tests for Cognitive Factors. Koran also used programmed materials dealing with selected areas of elementary statistics and test interpretation. There were four forms of the programmed material --- two inductive and two deductive with differing frequencies of alternation. These were distributed randomly to the students.

Each student was given a test two weeks after the completion of the programmed material. An analysis of the data showed no significant differences in the time required to complete the material. However, subjects in the inductive

treatment made significantly more errors than those in the deductive treatment.

Becker (1967) investigated the interaction of two instructional treatments with two aptitude variables. His subjects were students enrolled in an Algebra I class in San Carlos, California. All students were given multiple choice tests to determine their mathematical and verbal aptitude. On the basis of these tests 35 matched pairs were obtained. Subjects in each matched pair were randomly assigned to treatments. The two treatments were programmed instruction --- one inductive and the other deductive in arrangement. The data collected were subjected to a multiple regression analysis. There were no significant differences.

Tanner (1968) studied the relative effectiveness of an expository treatment as compared to a discovery approach to teaching physical science. The experimental population consisted of 389 ninth-grade students enrolled in a general science course. These subjects were randomly assigned to three groups. One group received materials programmed in an expository-deductive format. The second group received materials programmed in a discovery-inductive format. The third group received materials containing the same program frames but in a random order. An analysis of posttest scores found no significant differences among the three groups.

Laboratory Studies

In the last five years, nearly all research of an experimental nature dealing with the laboratory approach to instruction has been undertaken by doctoral students. The intent of the studies has varied widely as have the populations sampled. Wilkinson (1970) was interested in determining whether the laboratory approach to teaching geometry to sixth graders would be more effective than the traditional teacher-textbook approach. He used two experimental groups and one control group. One experimental group used manipulative materials and worksheets to guide them in collecting and generalizing their data. The second experimental group received verbal instructions, by means of tape cassettes, in addition to the written worksheets. All three groups were given pretests and posttests dealing with their attitude toward mathematics, achievement in geometry, and nonverbal intelligence. An analysis of the data showed no significant differences for the three groups in any of these areas.

In another study, Cohen (1970) investigated the relative effectiveness of the mathematics laboratory on underachieving seventh and eighth grade boys. Two groups of fourteen boys each with average or above-average ability, but with below-average achievement were used. One group was taught fraction concepts and computation with fractions using the traditional textbook-discussion approach. The

second group was taught the same material in a laboratory setting using manipulative devices and multi-sensory materials. A comparison of achievement scores, computational skills scores, and attitude scores for the two groups showed no significant differences.

Three studies dealt with the use of the laboratory approach at the college level. Phillips (1970) conducted a study at Oakland City College, Oakland City, Indiana, to determine the effect of the laboratory approach on the achievement and attitude of low-achiever mathematics students enrolled in a developmental mathematics course. All subjects were given a pretest and two posttests to determine attitude and achievement. One posttest was given at the end of the course while the second was given at the end of a second required mathematics course. The experimental group was taught the developmental mathematics by means of a mathematics laboratory. The control group received the traditional lecture type of instruction. Both groups received the traditional approach in the sequential course. An analysis of the data showed no significant difference in achievement for the two groups. The laboratory group did show a significant improvement in their attitude immediately following the laboratory experience but this gain was no longer apparent following their re-encounter with the traditional approach.

In the second study, Smith (1970) investigated the effectiveness of the laboratory in teaching abstract algebra to college students. He used two classes of college students

enrolled in a required course in abstract algebra. Each class was halved so that there were four groups with twelve students in each. The control group received four lecture sessions with no laboratory. The other three groups received varying laboratory sessions. One group had one lecture session and three laboratory sessions; the second had two lecture sessions and two laboratory sessions; the third group had three lecture sessions and one laboratory session. The laboratory experience dealt with the manipulation of concrete models relating to the materials taught in the lecture sessions. The material dealt with systems of numeration and bases other than ten. The analysis of the data indicated that those receiving laboratory experience scored significantly higher than the control group in both comprehension and retention.

The third study, conducted by Bluman (1971), was to determine whether the laboratory method of instruction in mathematics would be more effective than the traditional approach. For the purposes of this study, four intact classes of freshmen enrolled in a college-level introductory mathematics course were selected. Two classes acted as control groups and received the traditional instruction. The other two classes received their instruction by means of filmstrips, experiments, demonstrations, overhead projector, and problem sessions. Two teachers were used to teach these four classes. Each teacher had an experimental and a control group. The analysis of the data indicated that there was no

significant difference between the two treatments in either attitude or achievement. There was, however, a significant interaction between teacher and method.

In all of the above studies, the general purpose was to determine whether the laboratory approach should be used in place of the traditional approach. As stated in Chapter I this either-or approach is inappropriate in view of today's educational philosophy. Instead, we need to ascertain in what way the laboratory can best be employed to complement the traditional approach. It is this question which needs to be answered.

In researching the literature, two studies were found which closely resemble the present study. The first study was conducted by Reuss (1970). Reuss used three groups of biology students all receiving laboratory experiences. The control group used experiments employing the traditional deductive approach. One experimental group used experiments which were of the guided inductive type, while the other group used materials written in the open inductive style. All students were pretested on attitude and basic knowledge of the topic to be studied. Posttests were given. The data were analyzed with the class as the basic statistical unit and again taking ability into consideration. In all cases, there were no significant differences among the three approaches.

In the second study, Emslie (1971) sought to determine the relative effectiveness of two sequencing procedures

in the teaching of a unit on molecules and the atom. Method I was a laboratory-theory sequence while Method II was a theory-laboratory sequence. Method I was used with a sample of 99 fourth and sixth graders in one school while Method II was used with a sample of 158 fourth and sixth graders in a school in another district. The criterion variable was the score on a standardized test designed for use with a sixth grade science textbook. The data were analyzed using analysis of covariance with I. Q. and general science achievement scores as covariates. This analysis resulted in no significant differences for the two methods although the fourth graders appeared to score higher under the laboratory-theory approach.

Summary

In general the studies comparing the inductive with the deductive approach have been inconclusive. Both approaches seem to have merit and would lead one to believe that the mathematics laboratory could logically precede or follow a lecture-discussion presentation. Experimental testing of this assumption is needed, however.

The studies dealing with the laboratory approach generally found it to be as effective as more traditional approaches. These results should guarantee the laboratory method a slot in every educator's repertoire. The issue which has not been answered, at least for the mathematics

laboratory, is how most effectively to combine the laboratory approach with the traditional lecture-discussion approach. The studies by Reuss (1970) and Emslie (1971) illustrate that this concern is shared by other sciences. Although their studies found no significant differences for different sequencing patterns, they have served to make us aware of the need for further research in other areas and at other grade levels. In the present study, the role of the mathematics laboratory at the college level has been investigated. It is hoped that this study will contribute additional information about the mathematics laboratory and its relation to more traditional modes of instruction.

CHAPTER III

THE EXPERIMENTAL DESIGN

Statement of Hypotheses

As the preceding chapter shows, there is a dearth of research dealing with the laboratory as an adjunct to more traditional modes of instruction. Although the two studies that dealt with this question had no significant results, there were trends within intelligence levels which indicate that further research might be informative.

A factor which was not considered in either of these studies was personality type. It is conceivable that the inductive nature of a laboratory experience might cause cognitive dissidence with certain personality types and hence have an effect on the results of the study.

In the present study, personality type and achievement level will both be taken into consideration and their effects, if any, determined. In order to do this, the following null hypotheses will be investigated:

H1. There is no significant difference between the mean posttest score of students in the exploratory-discovery group and the mean posttest score of students in the verification-application group.

H2. There is no significant difference between the mean posttest score of students in the exploratory-discovery group and the mean posttest score of students in the combination group.

H3. There is no significant difference between the mean posttest score of students in the verification-application group and the mean posttest score of students in the combination group.

H4. There is no significant difference between the mean posttest score of sensing students and the mean posttest score of intuitive students.

H5. There is no significant difference between the mean posttest score of high-achieving students in the exploratory-discovery group and the mean posttest score of high-achieving students in the verification-application group.

H6. There is no significant difference between the mean posttest score of high-achieving students in the exploratory-discovery group and the mean posttest score of high-achieving students in the combination group.

H7. There is no significant difference between the mean posttest score of high-achieving students in the verification-application group and the mean posttest score of high-achieving students in the combination group.

H8. There is no significant difference between the mean posttest score of average-achieving students in the exploratory-discovery group and the mean posttest score of average-achieving students in the verification-application group.

H9. There is no significant difference between the mean posttest score of average-achieving students in the exploratory-discovery group and the mean posttest score of average-achieving students in the combination group.

H10. There is no significant difference between the mean posttest score of average-achieving students in the verification-application group and the mean posttest score of average-achieving students in the combination group.

H11. There is no significant difference between the mean posttest score of low-achieving students in the exploratory-discovery group and the mean

posttest score of low-achieving students in the verification-application group.

H12. There is no significant difference between the mean posttest score of low-achieving students in the exploratory-discovery group and the mean posttest score of low-achieving students in the combination group.

H13. There is no significant difference between the mean posttest score of low-achieving students in the verification-application group and the mean posttest score of low-achieving students in the combination group.

H14. There is no significant difference between the mean posttest score of sensing students in the exploratory-discovery group and the mean posttest score of sensing students in the verification-application group.

H15. There is no significant difference between the mean posttest score of sensing students in the exploratory-discovery group and the mean posttest score of sensing students in the combination group.

H16. There is no significant difference between the mean posttest score of sensing students in the verification-application group and the mean posttest score of sensing students in the combination group.

H17. There is no significant difference between the mean posttest score of intuitive students in the exploratory-discovery group and the mean posttest score of intuitive students in the verification-application group.

H18. There is no significant difference between the mean posttest score of intuitive students in the exploratory-discovery group and the mean posttest score of intuitive students in the combination group.

H19. There is no significant difference between the mean posttest score of intuitive students in the verification-application group and the mean posttest score of intuitive students in the combination group.

H20. There is no significant difference between the mean posttest score of high-achieving sensing

students in the exploratory-discovery group and the mean posttest score of high-achieving sensing students in the verification-application group.

H21. There is no significant difference between the mean posttest score of high-achieving sensing students in the exploratory-discovery group and the mean posttest score of high-achieving sensing students in the combination group.

H22. There is no significant difference between the mean posttest score of high-achieving sensing students in the verification-application group and the mean posttest score of high-achieving sensing students in the combination group.

H23. There is no significant difference between the mean posttest score of average-achieving sensing students in the exploratory-discovery group and the mean posttest score of average-achieving sensing students in the verification-application group.

H24. There is no significant difference between the mean posttest score of average-achieving sensing students in the exploratory-discovery group and the mean posttest score of average-achieving sensing students in the combination group.

H25. There is no significant difference between the mean posttest score of average-achieving sensing students in the verification-application group and the mean posttest score of average-achieving sensing students in the combination group.

H26. There is no significant difference between the mean posttest score of low-achieving sensing students in the exploratory-discovery group and the mean posttest score of low-achieving sensing students in the verification-application group.

H27. There is no significant difference between the mean posttest score of low-achieving sensing students in the exploratory-discovery group and the mean posttest score of low-achieving sensing students in the combination group.

H28. There is no significant difference between the mean posttest score of low-achieving sensing students in the verification-application group and the mean posttest score of low-achieving sensing students in the combination group.

H29. There is no significant difference between the mean posttest score of high-achieving intuitive students in the exploratory-discovery group and the mean posttest score of high-achieving intuitive students in the verification-application group.

H30. There is no significant difference between the mean posttest score of high-achieving intuitive students in the exploratory-discovery group and the mean posttest score of high-achieving intuitive students in the combination group.

H31. There is no significant difference between the mean posttest score of high-achieving intuitive students in the verification-application group and the mean posttest score of high-achieving intuitive students in the combination group.

H32. There is no significant difference between the mean posttest score of average-achieving intuitive students in the exploratory-discovery group and the mean posttest score of average-achieving intuitive students in the verification-application group.

H33. There is no significant difference between the mean posttest score of average-achieving intuitive students in the exploratory-discovery group and the mean posttest score of average-achieving intuitive students in the combination group.

H34. There is no significant difference between the mean posttest score of average-achieving intuitive students in the verification-application group and the mean posttest score of average-achieving intuitive students in the combination group.

H35. There is no significant difference between the mean posttest score of low-achieving intuitive students in the exploratory-discovery group and the mean posttest score of low-achieving intuitive students in the verification-application group.

H36. There is no significant difference between the mean posttest score of low-achieving intuitive students in the exploratory-discovery group and the mean posttest score of low-achieving intuitive students in the combination group.

H37. There is no significant difference between the mean posttest score of low-achieving intuitive

students in the verification-application group and the mean posttest score of low-achieving intuitive students in the combination group.

Description of Procedures and Design

The design of the present study can best be classified as the nonequivalent control group design as described by Campbell and Stanley (1963). There is not, however, a control group as such since all groups involved received a treatment. More specifically, the study is a 3x3x2 factorial experiment. The three experimental factors are the sequencing pattern (exploratory-discovery, verification-application, combination), achievement status (high, average, low) and personality type (sensing, intuitive). The criterion measure is the error score on a posttest on ratio and similarity with the error score on a pretest on ratio and similarity as a covariate.

For the purposes of this study, seven classes of freshmen students enrolled in a required mathematics course at Santa Fe Junior College were selected and constituted the experimental population. These seven classes were selected on the basis of the willingness of the instructors to participate in the study and the fact that each instructor had at least two classes at approximately the same time of day. There were three instructors involved in the study --- two instructors had two classes apiece while the third had three classes. Five of the classes were during the day and met

for 95 minutes at each session. The remaining two classes were at night and met for two hours at a time.

Since it was not possible to assign students randomly to these seven classes, the classes were randomly assigned to treatments. The two instructors with two classes each had two of the three treatments but not the same two. The third instructor had all three treatments. Diagrammatically, the design would look something like the following:

TREATMENT	TEACHER A	TEACHER B	TEACHER C
I		X	X
II	X		X
III	X	X	X

Each of the seven classes received laboratory experiences in conjunction with their study of ratio and similarity. The variable was in the sequencing of the laboratory experience with the class discussion. The one treatment group received their laboratory experiences before the classroom instruction, hereafter referred to as the exploratory-discovery method. The laboratory experience consisted of a series of guided experiments on ratio and similarity. The exploratory-discovery group received sixty minutes of laboratory experience followed immediately by thirty minutes of class presentation on the principles observed in the experiments.

The second treatment group, hereafter referred to as the verification-application group, received thirty minutes of class presentation followed immediately by sixty minutes of laboratory experience.

The third group, hereafter referred to as the combination group, received thirty minutes of laboratory experiences followed by thirty minutes of discussion, which was followed by another thirty minutes of laboratory experiences.

In August, 1972, before undertaking their study of ratio and similarity, all three groups were administered the Myers-Briggs Type Indicator and a pretest on ratio and similarity to determine their background knowledge on this topic. (The pretest-posttest was designed by the author and will be described in the next section.) They were also given the opportunity to perform some laboratory experiments dealing with area of a circle and the calculation of pi so that they would be familiar with this method of instruction. After the unit on ratio and similarity was completed, each group was given a brief questionnaire dealing with their personal reaction to the laboratory experience and a posttest.

The seven classes used had a total enrollment of 129 students. For a student to be included in the study it was necessary to have four pieces of data on him. They were a pretest score, a posttest score, a Myers-Briggs Type Indicator classification, and an overall grade point average for his work at Santa Fe Junior College. Deletion of those subjects with incomplete data left a sample population of 94

subjects. These 94 subjects were categorized by treatment, achievement level, and personality type.

The basis for determining a student's achievement level was his overall grade point average (GPA) at Santa Fe Junior College. At this junior college only four letter grades were in use --- A, B, C, and W. A grade of A was worth four points per semester hour of credit earned; a grade of B was worth three points per semester hour of credit earned; a grade of C was worth two points per semester hour of credit earned; and a grade of W, which normally is not used in the calculation of the GPA, was assigned one point per semester hour of credit attempted. If a student's GPA was greater than or equal to 3.35, he was termed a high-achiever. If the GPA was between 2.65 and 3.35, he was termed an average-achiever. If the GPA was less than or equal to 2.65, he was classified a low-achiever. The distribution of the subjects taking into account treatment and achievement level is shown in Table I.

TABLE I: Subject Distribution by Treatment and Achievement Level

	High Achievers	Average Achievers	Low Achievers	Total
Exploratory-Discovery	14	10	5	29
Verification-Application	8	4	9	21
Combination	31	9	4	44
Total	53	23	18	94

In order to divide the students into two broad personality types, all subjects were administered the Myers-Briggs Type Indicator. This test measures four dichotomous dimensions of the personality. They are: judgment-perception, thinking-feeling, sensing-intuition, and extraversion-introversion. For the purposes of this study, only the sensing-intuition dimension was used. (This test will be described in detail in the next section.) On the basis of this test, students were classified as sensing, that is, using data perceived through the senses to draw conclusions or make decisions; or as intuitive, that is, tending to rely upon imagination and inspiration for decisions. The distribution of the subjects taking into account treatment and personality type is shown in Table II, while Table III gives the distribution using the factors of achievement level and personality type.

TABLE II: Subject Distribution by Treatment and Personality Type

	Sensing	Intuitive	Total
Exploratory-Discovery	18	11	29
Verification-Application	13	8	21
Combination	19	25	44
Total	50	44	94

TABLE III: Subject Distribution by Achievement Level and Personality Type

	Sensing	Intuitive	Total
High-Achiever	26	27	53
Average-Achiever	12	11	23
Low-Achiever	12	6	18
Total	50	44	94

Instrumentation

This section is devoted to an examination of the experimental materials and two test instruments utilized in the present study.

Myers-Briggs Type Indicator

As indicated in the preceding section, the subjects were classified by personality types by means of the Myers-Briggs Type Indicator, Form F (grades 9-16). This is a

forced choice, self-report inventory consisting of 166 questions and is designed to be used with normal subjects. It is administered in a group setting and requires approximately fifty-five minutes to complete.

The test purports to measure the following four dichotomous dimensions: judgment-perception, thinking-feeling, sensing-intuition, and extraversion-introversion. Each student's answer sheet must be graded eight times to obtain a preference for each of these dimensions. An adjusted score is determined through the use of prepared tables found in the Myers-Briggs Type Indicator Manual. This adjusted score gives not only a preference but also an indication of the strength of that preference. Since the present study dealt principally with a student's reasoning ability, it was decided to use only the sensing-intuition dimension. This dimension has been characterized in the following way. "When people prefer sensing, they find too much of interest in the actuality around them to spend much energy listening for ideas out of nowhere. When people prefer intuition, they are too much interested in all the possibilities that occur to them to give a whole lot of notice to the actualities" (Myers 1962, p. 51).

The Myers-Briggs Type Indicator has been developed over a twenty year period. The developers assert that it is based on the Jungian theory of type, but the true dichotomy of the dimensions has been questioned by a number of psychologists. To determine content validity, split-half

reliability coefficients, corrected by the Spearman-Brown prophecy formula, were calculated for each dimension at different grade levels. The sensing-intuition scale had a reliability coefficient of 0.87 for college students.

Pretest-Posttest

In June 1972, twenty behavioral objectives on the topic of ratio and similarity were developed. These were submitted to a panel of judges consisting of three junior college mathematics teachers. The panel assessed the objectives and found them to be appropriate for both the topic of study and the grade level. From these objectives a pretest-posttest designed to assess the subjects' knowledge of ratio and similarity was developed. One question was prepared for each objective. The test was submitted to the same panel and adjudged to be appropriate for the stated objectives.

The test was administered to the students in a section of the required mathematics course which was not to be involved in the study. There were twenty students in the class. The results of this trial run were subjected to a difficulty test using the following criterion: If X represents the number of correct responses to a particular question, then the question is judged to be acceptable only if $.10N < X < .90N$, where N represents the total number of students taking the test. According to this formula, all questions were acceptable.

The test was then administered to all subjects in

the seven experimental classes at the beginning and end of the unit on ratio and similarity. Copies of the performance objectives and pretest-posttest may be found in Appendix A.

Experiment Materials

All of the experiments used in this study were taken from The Laboratory Approach to Mathematics by Kidd, Myers and Cilley or from unpublished materials developed by Kenneth P. Kidd. Some modifications in the materials used were made. Copies of the experiments may be found in Appendix B.

Statistical Treatment

The data gathered in the present study were analyzed using the system of multiple linear regression. A computer program called MANOVA was employed to compute the error sum of squares and F-statistics for all main effects and interactions. The criterion variable was the posttest error scores while the pretest error scores were used as a covariate. The calculated F-values were used to determine whether to accept or reject the null hypotheses at a predetermined level of confidence. Scheffé's Method was also used to determine whether reductions in error scores were significant.

CHAPTER IV

ANALYSIS OF DATA

The first three hypotheses involve a comparison of the exploratory-discovery method, the verification-application method and the combination method without regard to achievement level or personality type. The mean error scores on the pretest and posttest for these hypotheses are found in Table IV. Table V is the analysis of covariance table for the entire study.

TABLE IV: Mean Error Scores for Subjects in the Exploratory-Discovery Group, the Verification-Application Group and Combination Group on the Pretest and Posttest

	Pretest	Posttest	Difference
Exploratory-Discovery	9.897	5.379	4.518
Verification-Application	8.048	5.190	2.858
Combination	7.636	4.341	3.295

H1. There is no significant difference between the mean posttest score of students in the exploratory-discovery group and the mean posttest score of students in the verification-application group.

TABLE V: Analysis of Covariance

Source	Sum of Squares	DF	Mean Squares	F	Significance Level
Within Cells	299.723	75	3.996		
Regression	89.750	1	89.750	22.458	$p < 0.001$
Method	8.708	2	4.354	1.089	$p < 0.342$
Achievement	16.061	2	8.031	2.010	$p < 0.141$
Personality	21.249	1	21.249	5.317	$p < 0.024$
Method x Achievement	23.996	4	5.999	1.501	$p < 0.210$
Method x Personality	4.616	2	2.308	0.578	$p < 0.564$
Achievement x Personality	2.988	2	1.494	0.374	$p < 0.689$
Method x Achievement x Personality	15.705	4	3.926	0.982	$p < 0.422$

H2. There is no significant difference between the mean posttest score of students in the exploratory-discovery group and the mean posttest score of students in the combination group.

H3. There is no significant difference between the mean posttest score of students in the verification-application group and the mean posttest score of students in the combination group.

Hypotheses H1-H3 state that there are no differences among the mean error scores of subjects in the exploratory-discovery group, the verification-application group and the combination group. If these hypotheses are in fact true, then differences as large or larger than those observed could occur by chance 14.1 percent of the time. The F-ratio for method in Table V is less than that required for significance at the 0.05 confidence level, and hence none of the null hypotheses H1-H3 can be rejected. This indicates that there is no significant difference among the mean error scores for the three methods. However, use of Scheffé's Method to compare the differences between the posttest and pretest mean error scores indicates that the exploratory-discovery group achieved a significantly greater reduction in mean error score than either of the other methods. This is shown by the 95 percent confidence intervals found in Table VI.

TABLE VI: 95 Percent Confidence Intervals for Comparisons Among the Exploratory-Discovery Group, the Verification-Application Group and the Combination Group

Method	Contrasts	Confidence Interval
Exploratory-Discovery minus Verification-Application	1.660	0.632 to 2.688
Exploratory-Discovery minus Combination	1.223	0.257 to 2.189
Verification-Application minus Combination	-0.437	-1.508 to 0.634

Hypothesis H4 involves a comparison of sensing subjects and intuitive subjects. The mean error scores for this hypothesis are given in Table VII.

TABLE VII: Mean Error Scores of Sensing Subjects and Intuitive Subjects on the Pretest and Posttest

	Pretest	Posttest	Difference
Sensing Subjects	8.700	5.420	3.280
Intuitive Subjects	8.114	4.205	3.909

H4. There is no significant difference between the mean posttest score of sensing students and the mean posttest score of intuitive students.

Hypothesis H4 states that there are no differences between the mean error scores of subjects who have been categorized as sensing and those who have been categorized as intuitive. If this hypothesis is in fact true, then

differences as large or larger than those observed could occur by chance 2.4 percent of the time. The F-ratio for personality in Table V exceeds that required for significance at the 0.05 confidence level, and hence the null hypothesis H4 can be rejected. This means that the sensing students did significantly better on the posttest than the intuitive students. This would imply that laboratory experiences are more meaningful for those students who rely upon their senses than for those who rely upon their feelings and imagination.

Hypotheses H5-H13 involve the comparison of the three methods of instruction within achievement levels. Table VIII shows the mean pretest scores and mean posttest scores for high-achievers in the exploratory-discovery group, the verification-application group and the combination group.

TABLE VIII: Mean Error Scores for High-Achievers in the Exploratory-Discovery Group, the Verification-Application Group and the Combination Group on the Pretest and Posttest

	Pretest	Posttest	Difference
Exploratory-Discovery	8.214	5.000	3.214
Verification-Application	6.000	3.750	2.250
Combination	7.000	3.774	3.226

The mean pretest scores and mean posttest scores for average-achievers in the exploratory-discovery group, the verification-application group and the combination group may be found in Table IX.

TABLE IX: Mean Error Scores for Average-Achievers in the Exploratory-Discovery Group, the Verification-Application Group and the Combination Group on the Pretest and Posttest

	Pretest	Posttest	Difference
Exploratory-Discovery	12.600	5.800	6.800
Verification-Application	8.250	6.250	2.000
Combination	8.556	6.000	2.556

Table X displays the mean pretest scores and mean posttest scores for low-achievers in the exploratory-discovery group, the verification-application group and the combination group.

TABLE X: Mean Error Scores for Low-Achievers in the Exploratory-Discovery Group, the Verification-Application Group and the Combination Group on the Pretest and Posttest

	Pretest	Posttest	Difference
Exploratory-Discovery	9.200	5.600	2.600
Verification-Application	9.778	6.000	3.778
Combination	10.500	5.000	5.500

H5. There is no significant difference between the mean posttest score of high-achieving students in the exploratory-discovery group and the mean posttest score of high-achieving students in the verification-application group.

H6. There is no significant difference between the mean posttest score of high-achieving students in the

exploratory-discovery group and the mean posttest score of high-achieving students in the combination group.

H7. There is no significant difference between the mean posttest score of high-achieving students in the verification-application group and the mean posttest score of high-achieving students in the combination group.

H8. There is no significant difference between the mean posttest score of average-achieving students in the exploratory-discovery group and the mean posttest score of average-achieving students in the verification-application group.

H9. There is no significant difference between the mean posttest score of average-achieving students in the exploratory-discovery group and the mean posttest score of average-achieving students in the combination group.

H10. There is no significant difference between the mean posttest score of average-achieving students in the verification-application group and the mean posttest score of average-achieving students in the combination group.

H11. There is no significant difference between the mean posttest score of low-achieving students in the exploratory-discovery group and the mean posttest score of low-achieving students in the verification-application group.

H12. There is no significant difference between the mean posttest score of low-achieving students in the exploratory-discovery group and the mean posttest score of low-achieving students in the combination group.

H13. There is no significant difference between the mean posttest score of low-achieving students in the verification-application group and the mean posttest score of low-achieving students in the combination group.

Hypotheses H5-H7 state that there are no significant differences among the mean error scores on the posttest of high-achievers in the three laboratory sequencing treatments. Hypotheses H8-H10 state that there are no significant

differences among the mean error scores on the posttest of average-achievers in the three treatment groups. Hypotheses H11-H13 state that there are no significant differences among the mean error scores on the posttest of low-achievers in the three treatment groups. The F-values found in Table V indicate that both main effects are not significant at the 0.05 level of confidence. The F-ratio of 1.501 for method-achievement interaction also is less than that required for significance at the 0.05 confidence level. Therefore, we cannot reject the hypothesis of no interaction. This also means that we can reject none of the hypotheses H5-H13.

Use of Scheffé's Method to compare the differences between the posttest and pretest mean error scores for average-achievers indicates that the exploratory-discovery group achieved a significantly greater reduction in mean error scores than either of the other methods. This is shown by the 95 percent confidence intervals found in Table XI.

TABLE XI: 95 Percent Confidence Intervals for Comparisons Among the Average-Achievers in the Exploratory-Discovery Group, the Verification-Application Group and the Combination Group

Method	Contrasts	Confidence Interval
Exploratory-Discovery minus Verification-Application	4.800	2.411 to 7.189
Exploratory-Discovery minus Combination	4.224	2.369 to 6.079
Verification-Application minus Combination	-0.556	-2.982 to 1.870

Use of Scheffé's Method to compare the differences between the posttest and pretest mean error scores for low-achievers indicates that the combination group achieved a significantly greater reduction in mean error scores than the exploratory-discovery group. This is shown by the 95 percent confidence intervals found in Table XII.

TABLE XII: 95 Percent Confidence Intervals for Comparisons Among the Low-Achievers in the Exploratory-Discovery Group, the Verification-Application Group and the Combination Group

Method	Contrasts	Confidence Interval
Exploratory-Discovery minus Verification-Application	-1.178	-3.431 to 1.074
Exploratory-Discovery minus Combination	-2.900	-5.609 to -0.191
Verification-Application minus Combination	-2.222	-4.648 to 0.204

Hypotheses H14-H19 involve the comparison of the three methods of instruction within personality types. Table XIII shows the mean pretest scores and mean posttest scores for sensing students in the exploratory-discovery group, the verification-application group and the combination group.

TABLE XIII: Mean Error Scores for Sensing Subjects in the Exploratory-Discovery Group, the Verification-Application Group and the Combination Group on the Pretest and Posttest

	Pretest	Posttest	Difference
Exploratory-Discovery	10.611	6.167	4.444
Verification-Application	7.154	5.077	2.077
Combination	7.947	4.947	3.000

The mean pretest scores and mean posttest scores for intuitive students in the exploratory-discovery group, the verification-application group and the combination group may be found in Table XIV.

TABLE XIV: Mean Error Scores for Intuitive Subjects in the Exploratory-Discovery Group, the Verification-Application Group and the Combination Group on the Pretest and Posttest

	Pretest	Posttest	Difference
Exploratory-Discovery	8.727	4.091	4.636
Verification-Application	9.500	5.375	4.125
Combination	7.400	3.880	3.520

H14. There is no significant difference between the mean posttest score of sensing students in the exploratory-discovery group and the mean posttest score of sensing students in the verification-application group.

H15. There is no significant difference between the mean posttest score of sensing students in the exploratory-discovery group and the mean posttest score of sensing students in the combination group.

H16. There is no significant difference between the mean posttest score of sensing students in the verification-application group and the mean posttest score of sensing students in the combination group.

H17. There is no significant difference between the mean posttest score of intuitive students in the exploratory-discovery group and the mean posttest score of intuitive students in the verification-application group.

H18. There is no significant difference between the mean posttest score of intuitive students in the exploratory-discovery group and the mean posttest score of intuitive students in the combination group.

H19. There is no significant difference between the mean posttest score of intuitive students in the verification-application group and the mean posttest score of intuitive students in the combination group.

Hypotheses H14-H16 state that there are no significant differences among the mean error scores on the posttest of sensing subjects in the three laboratory sequencing treatments. Hypotheses H17-H19 state that there are no significant differences among the mean error scores on the posttest of intuitive subjects in the three treatment groups. The F-ratios found in Table V indicate that only the main effect of personality is significant at the 0.05 level of confidence. The F-ratio of 0.578 for method-personality interaction is less than that needed for significance at the 0.05 confidence level. Therefore, we cannot reject the hypothesis of no interaction. This also means that we can reject none of the hypotheses H14-H19.

Use of Scheffé's Method to compare the difference between the posttest and pretest mean error scores for sensing students indicates that the exploratory-discovery group achieved a significantly greater reduction in mean error scores than either of the other methods. This is shown by the 95 percent confidence intervals given in Table XV.

TABLE XV: 95 Percent Confidence Intervals for Comparisons Among the Sensing Subjects in the Exploratory-Discovery Group, the Verification-Application Group and the Combination Group

Method	Contrasts	Confidence Interval
Exploratory-Discovery minus Verification-Application	2.367	0.660 to 4.074
Exploratory-Discovery minus Combination	1.444	0.116 to 2.772
Verification-Application minus Combination	-0.923	-2.376 to 0.530

Hypotheses H20-H37 involve the comparison of the three methods of instruction within achievement levels taking personality type into account. Table XVI shows the mean pretest scores and mean posttest scores for sensing high-achievers in the exploratory-discovery group, the verification-application group and the combination group.

TABLE XVI: Mean Error Scores for Sensing High-Achievers in the Exploratory-Discovery Group, the Verification-Application Group and the Combination Group on the Pretest and Posttest

	Pretest	Posttest	Difference
Exploratory-Discovery	7.143	5.571	1.572
Verification-Application	4.600	4.000	0.600
Combination	6.857	4.071	2.786

The mean pretest scores and mean posttest scores for sensing average-achievers in the exploratory-discovery group, the verification-application group and the combination group may be found in Table XVII.

TABLE XVII: Mean Error Scores for Sensing Average-Achievers in the Exploratory-Discovery Group, the Verification-Application Group, and the Combination Group on the Pretest and Posttest

	Pretest	Posttest	Difference
Exploratory-Discovery	14.143	6.571	7.572
Verification-Application	3.500	5.000	-1.500
Combination	10.333	8.667	1.666

Table XVIII displays the mean pretest scores and mean posttest scores of sensing low-achievers in the exploratory-discovery group, the verification-application group and the combination group.

TABLE XVIII: Mean Error Scores for Sensing Low-Achievers in the Exploratory-Discovery Group, the Verification-Application Group and the Combination Group on the Pretest and Posttest

	Pretest	Posttest	Difference
Exploratory-Discovery	10.500	6.500	4.000
Verification-Application	10.500	6.000	4.500
Combination	12.000	5.500	6.500

Table XIX shows the mean pretest scores and mean posttest scores for intuitive high-achievers in the exploratory-discovery group, the verification-application group and the combination group.

TABLE XIX: Mean Error Scores for Intuitive High-Achievers in the Exploratory-Discovery Group, the Verification-Application Group and the Combination Group on the Pretest and Posttest

	Pretest	Posttest	Difference
Exploratory-Discovery	9.286	4.429	4.857
Verification-Application	8.333	3.333	5.000
Combination	7.118	3.529	3.589

The mean pretest scores and mean posttest scores for intuitive average-achievers in the exploratory-discovery group, the verification-application group and the combination group may be found in Table XX.

TABLE XX: Mean Error Scores for Intuitive Average-Achievers in the Exploratory-Discovery Group, the Verification-Application Group and the Combination Group on the Pretest and Posttest

	Pretest	Posttest	Difference
Exploratory-Discovery	9.000	4.000	5.000
Verification-Application	13.000	7.500	5.500
Combination	7.667	4.667	3.000

Table XXI displays the mean pretest scores and mean posttest scores of intuitive low-achievers in the exploratory-discovery group, the verification-application group and the combination group.

TABLE XXI: Mean Error Scores for Intuitive Low-Achievers in the Exploratory-Discovery Group, the Verification-Application Group and the Combination Group on the Pretest and Posttest

	Pretest	Posttest	Difference
Exploratory-Discovery	4.000	2.000	2.000
Verification-Application	8.333	6.000	2.333
Combination	9.000	4.500	4.500

H20. There is no significant difference between the mean posttest score of high-achieving sensing students in the exploratory-discovery group and the mean posttest score of high-achieving sensing students in the verification-application group.

H21. There is no significant difference between the mean posttest score of high-achieving sensing students in the exploratory-discovery group and the mean posttest score of high-achieving sensing students in the combination group.

H22. There is no significant difference between the mean posttest score of high-achieving sensing students in the verification-application group and the mean posttest score of high-achieving sensing students in the combination group.

H23. There is no significant difference between the mean posttest score of average-achieving sensing students in the exploratory-discovery group and the mean posttest score of average-achieving sensing students in the verification-application group.

H24. There is no significant difference between the mean

posttest score of average-achieving sensing students in the exploratory-discovery group and the mean posttest score of average-achieving sensing students in the combination group.

H25. There is no significant difference between the mean posttest score of average-achieving sensing students in the verification-application group and the mean posttest score of average-achieving sensing students in the combination group.

H26. There is no significant difference between the mean posttest score of low-achieving sensing students in the exploratory-discovery group and the mean posttest score of low-achieving sensing students in the verification-application group.

H27. There is no significant difference between the mean posttest score of low-achieving sensing students in the exploratory-discovery group and the mean posttest score of low-achieving sensing students in the combination group.

H28. There is no significant difference between the mean posttest score of low-achieving sensing students in the verification-application group and the mean posttest score of low-achieving sensing students in the combination group.

H29. There is no significant difference between the mean posttest score of high-achieving intuitive students in the exploratory-discovery group and the mean posttest score of high-achieving intuitive students in the verification-application group.

H30. There is no significant difference between the mean posttest score of high-achieving intuitive students in the exploratory-discovery group and the mean posttest score of high-achieving intuitive students in the combination group.

H31. There is no significant difference between the mean posttest score of high-achieving intuitive students in the verification-application group and the mean posttest score of high-achieving intuitive students in the combination group.

H32. There is no significant difference between the mean posttest score of average-achieving intuitive students in the exploratory-discovery group and the mean posttest score of average-achieving intuitive students in the verification-application group.

H33. There is no significant difference between the mean posttest score of average-achieving intuitive students in the exploratory-discovery group and the mean posttest score of average-achieving intuitive students in the combination group.

H34. There is no significant difference between the mean posttest score of average-achieving intuitive students in the verification-application group and the mean posttest score of average-achieving intuitive students in the combination group.

H35. There is no significant difference between the mean posttest score of low-achieving intuitive students in the exploratory-discovery group and the mean posttest score of low-achieving intuitive students in the verification-application group.

H36. There is no significant difference between the mean posttest score of low-achieving intuitive students in the exploratory-discovery group and the mean posttest score of low-achieving intuitive students in the combination group.

H37. There is no significant difference between the mean posttest score of low-achieving intuitive students in the verification-application group and the mean posttest score of low-achieving intuitive students in the combination group.

Hypotheses H20-H22 state that there are no significant differences among the mean error scores on the posttest of high-achieving sensing subjects in the three laboratory sequencing treatments. Hypotheses H23-H25 assert that there are no significant differences among the mean error scores on the posttest of average-achieving sensing subjects in the three treatment groups. Hypotheses H26-H28 state that there are no significant differences among the mean error scores on the posttest of low-achieving sensing subjects in the three treatment groups.

Hypotheses H29-H31 state that there are no significant differences among the mean error scores on the posttest

of high-achieving intuitive subjects in the three laboratory sequencing treatments. Hypotheses H32-H34 assert that there are no significant differences among the mean error scores on the posttest of average-achieving intuitive subjects in the three treatment groups. Hypotheses H35-H37 state that there are no significant differences among the mean error scores on the posttest of low-achieving intuitive subjects in the three treatment groups.

The F-ratios found in Table V indicate that the main effect of personality is the only one which is significant at the 0.05 level of confidence. The F-ratio of 0.982 for method-achievement-personality interaction is less than that required for significance at the 0.05 level of confidence. Therefore, we cannot reject the hypothesis of no interaction. In addition, none of the hypotheses H20-H37 can be rejected.

Use of Scheffé's Method to compare the differences between the posttest and pretest mean error scores for sensing high-achievers, indicates that the combination group achieved a significantly greater reduction in mean error scores than the verification-application group. This is shown by the 95 percent confidence intervals found in Table XXII.

TABLE XXII: 95 Percent Confidence Intervals for Comparisons Among the Sensing High-Achievers in the Exploratory-Discovery Group, the Verification-Application Group and the Combination Group

Method	Contrasts	Confidence Interval
Exploratory-Discovery minus Verification-Application	0.972	-1.392 to 3.336
Exploratory-Discovery minus Combination	-1.214	-3.083 to 0.655
Verification-Application minus Combination	-2.186	-4.290 to -0.082

Use of Scheffé's Method to compare the differences between the posttest and pretest mean error scores for sensing average-achievers indicates that the exploratory-discovery group achieved a significantly greater reduction in mean error scores than either of the other groups. This is shown by the 95 percent confidence intervals found in Table XXIII.

TABLE XXIII: 95 Percent Confidence Intervals for Comparisons Among the Sensing Average-Achievers in the Exploratory-Discovery Group, the Verification-Application Group and the Combination Group

Method	Contrasts	Confidence Interval
Exploratory-Discovery minus Verification-Application	9.072	5.575 to 12.569
Exploratory-Discovery minus Combination	5.906	3.120 to 8.692
Verification-Application minus Combination	-3.166	-6.852 to 0.520

At the conclusion of the study each student was asked to complete a questionnaire designed to measure his reactions to the laboratory experience. These questionnaires have been tabulated according to instructional treatment. In Table XXIV is the tabulation for the exploratory-discovery group. In Table XXV, is the tabulation for the verification-application group while the tabulation for the combination group is given in Table XXVI. A few selected comments by students in each of these groups will be found in the next three sections. The comments were also tabulated by instructional treatment to give an indication of the frequency of the various comments. These will be found in Tables XXVII, XXVIII and XXIX. All three methods received both favorable and unfavorable comments but the combination group seemed to be the most popular.

Comments from the Exploratory-Discovery Group

1. "I really enjoyed the experiment because you see things different after you learn it especially the ratios and things like that ...."
2. "Excellent for students who have difficulty with theory."
3. "If someone is slow to grasp concepts, this method is really hard to grasp. This method is fine for someone who has a good background in something similar."
4. "Not enough time to complete all experiments."
5. "The experiments would have been good for a fourth grade class. As a college course they were terribly BORING."
6. "When I don't understand I quit."
7. "I think I could have learnt more with the aid of an instructor previewing the work."

Comments from the Verification-Application Group

1. "The experiments were fun. It was like a learning game. Took the boredom out of the classroom."

2. "I found doing the experiments fun, and learned a great deal from doing them."

3. "It seems to be a pretty good method for teaching this subject matter. It still could use some refinement."

4. "Experiments were well thought-out. They seemed a little lengthy, though and I feel like there didn't need to be so many of them."

5. "I felt the experiments were too easy and the same thing could have been taught quicker in a classroom lecture."

Comments for the Combination Group

1. "It seems to visualize math and make it more understanding. Easier to handle and appreciate. It also gives the student a chance to do work and exercise without fear of failing a test, trying. We need this approach more."

2. "Very good. I wish I could of done this type learning all the way through the math course."

3. "It beats listening to lectures type classes all period and its easy to figure somethings out better on your own."

4. "It all seemed too easy, more like a game than like math, although it did get the point across rather well."

5. "I feel being able to 'stick your hands into it' teaches you more than watching someone else 'have all the fun' by being shown. I was very enthusiastic about it!!"

6. "Try something else."

7. "Most people were confused and turned off by the experiments."

TABLE XXIV: Questionnaire Summary for the Exploratory-Discovery Group

	Poor	Fair	Average	Good	Excellent
1. How appropriate were the experiments for this grade level?	3	1	9	14	4
2. How effective was this method of teaching <u>you</u> ratio and similarity?	3	2	6	14	6

	Yes	No
3. Was the time allowed to perform the experiments sufficient?	19	12
4. Would you recommend this method of teaching other units?	20	11
5. If this approach was available on a limited basis would you recommend it to a friend?	16	14
6. Did the experiments motivate you?	16	14
7. Did the experiments hold your interest?	18	13

TABLE XXV: Questionnaire Summary for the Verification-Application Group

	Poor	Fair	Average	Good	Excellent
1. How appropriate were the experiments for this grade level?	0	7	11	10	2
2. How effective was this method of teaching <u>you</u> ratio and similarity?	2	3	4	11	10
		Yes		No	
3. Was the time allowed to perform the experiments sufficient?		17		11	
4. Would you recommend this method of teaching other units?		20		9	
5. If this approach was available on a limited basis would you recommend it to a friend?		22		8	
6. Did the experiments motivate you?		22		8	
7. Did the experiments hold your interest?		16		9	

TABLE XXVI: Questionnaire Summary for the Combination Group

		Poor	Fair	Average	Good	Excellent
1.	How appropriate were the experiments for this grade level?	1	8	9	21	3
2.	How effective was this method of teaching <u>you</u> ratio and similarity?	5	7	7	13	11
			Yes		No	
3.	Was the time allowed to perform the experiments sufficient?		27		15	
4.	Would you recommend this method of teaching other units?		30		13	
5.	If this approach was available on a limited basis would you recommend it to a friend?		28		14	
6.	Did the experiments motivate you?		28		11	
7.	Did the experiments hold your interest?		26		12	

TABLE XXVII: Summary of Comments for the Exploratory-Discovery Group

	Frequency	Percent
Too easy and/or boring	4	12.9
Too difficult and/or not clear	7	22.6
Interesting and valuable	5	16.1
No comment	15	48.4

TABLE XXVIII: Summary of Comments for the Verification-Application Group

	Frequency	Percent
Too easy and/or boring	5	17.2
Too difficult and/or not clear	0	0.0
Interesting and valuable	5	17.2
No comment	19	65.6

TABLE XXIX: Summary of Comments for the Combination Group

	Frequency	Percent
Too easy and/or boring	3	7.1
Too difficult and/or not clear	4	9.5
Interesting and valuable	11	26.2
No comment	24	57.2

CHAPTER V

SUMMARY, CONCLUSIONS, LIMITATIONS, AND IMPLICATIONS

Summary

The purpose of this study was to investigate the relative effectiveness of the mathematics laboratory when used in three different ways in conjunction with a traditional lecture-discussion approach to teach community college freshmen enrolled in a required mathematics course. The laboratory experience was used as an introduction to a topic, as a reinforcer, or as both. All subjects were classified by achievement level and personality type, as determined by the Myers-Briggs Type Indicator, so that the interaction of these factors with the various laboratory approaches could be assessed. Previous research in scientific fields other than mathematics had studied the question of sequencing the laboratory experience with traditional teaching techniques but had found no significant results. These studies had not, however, taken achievement and/or personality into account. Because of the current popularity of the laboratory approach to teaching mathematics at all grade levels, it was decided to examine the issue of sequencing at the college level.

Ratio and similarity was selected as the unit to be taught using the combination of laboratory experiments and class discussion. This topic was selected for two reasons. First, several years of teaching had shown it to be a topic which was not familiar to the vast majority of college freshmen. Second, twenty tested and refined experiments on this topic were available.

A pretest-posttest was developed to determine a subject's knowledge of ratio and similarity. This test was submitted to a panel of judges for evaluation as to content validity and appropriateness for the grade level. It was also pilot tested on a class of freshmen mathematics students at Santa Fe Junior College. Subsequently, the pretest and Myers-Briggs Type Indicator were administered to seven mathematics classes of college freshmen who had been selected to participate in the study. These seven classes were randomly assigned to one of three laboratory sequencing patterns. Some groups received their laboratory experiences before the class discussion; some received their laboratory experience after the class discussion; and some received laboratory experiences both before and after the class discussion. All groups had the same length of time in the laboratory. At the end of the unit, the test on ratio and similarity was again administered. The error scores on the pretest were used as a covariate while the posttest scores served as the criterion variable. Prior to analyzing the data, a college grade point average was determined for each

subject and used to classify him as a high-achiever, average-achiever, or low-achiever. Only those students who had taken the pretest, posttest and Myers-Briggs Type Indicator were actually used in the study. There were 94 such students and these formed the sample for the study. Each student was classified according to the method of instruction he received, his achievement level and his personality type as determined by the sensing-intuition scale of the Myers-Briggs Type Indicator. This divided the sample population into eighteen subcategories varying in size from one to seventeen members.

The mean error scores for the eighteen groups were compared in a 3x3x2 factorial design using multiple linear regression techniques. A computer program called MANOVA was used to perform an analysis of covariance. The calculated F-ratios were used in testing the 37 null hypotheses. Scheffé's Method was used to determine the significance of reductions in mean error scores.

Conclusions

The following conclusions may be drawn from the study:

1. In a comparison of the exploratory-discovery group, the verification-application group and the combination group there were no significant differences among the mean posttest error scores. However, the exploratory-discovery

group achieved a significantly greater reduction in mean error score than either of the other groups.

2. In a comparison of sensing subjects to intuitive subjects, without regard to achievement level or method, the sensing subjects did significantly better than the intuitive students. This would imply that laboratory experiences are more meaningful to sensing students than to intuitive students.
3. In a comparison of the three methods of instruction within the high-achiever category without regard to personality type, there were no significant differences among the mean posttest error scores.
4. In a comparison of the three methods of instruction within the average-achiever category without regard to personality type, there were no significant differences among the mean posttest error scores. The subjects in the exploratory-discovery group did, however, achieve a significantly greater reduction in mean error score than either of the other groups.
5. In a comparison of the three methods of instruction within the low-achiever category without regard to personality type, there were no significant differences among the mean posttest error scores but the combination group did attain a significantly greater reduction in mean error score than the exploratory-discovery group.
6. In a comparison of the three methods of instruction for sensing subjects, there were no significant differences

among the mean posttest error scores. However, subjects in the exploratory-discovery group achieved a significantly greater reduction in mean error score than either of the other groups.

7. In a comparison of the three methods of instruction for intuitive subjects, there were no significant differences among the mean posttest error scores.
8. The interaction of method, achievement level and personality type did not make a statistically significant difference in error scores on the posttest. The sensing high-achievers in the combination group achieved a significantly greater reduction in mean error score than the verification-application group. The sensing average-achievers in the exploratory-discovery group achieved a significantly greater reduction in mean error score than either of the other groups.

In general, the results of the present study support the findings of Keuss (1970) and Emslie (1971). Although there were no significant differences among the posttest error scores for the three sequencing patterns, the significant reductions in mean error scores do have some educational implications. Significant reductions in the mean error score were most frequently attained by the exploratory-discovery group with the combination group a near second. The higher percentage of favorable comments for the combination group would suggest it is the better method in most situations. In addition, this method allows for the greatest diversity

among the students in the class. Those students who learn well on their own have an opportunity to discover new concepts for themselves and to receive almost immediate confirmation from the teacher. The additional laboratory experiences give further support to their findings. For those students who have difficulty abstracting or generalizing, the first laboratory experience may be frustrating and of limited value. The laboratory experience after the class discussion does, however, provide the opportunity to physically verify what has been taught in the classroom.

Limitations

Some of the limitations of the study are as follows:

1. Each teacher did not use all three methods of instruction. This means that teacher-method interaction could not be checked. It was assumed in the present study that the teacher effect would be nonsignificant.
2. The unit on ratio and similarity lasted for only two and a half weeks. This may have effected the mean error scores of low-achievers who possibly were slower in adjusting to the new technique and in performing the experiments.
3. The concentration of the research on a single unit of study may mean that the results are not valid for a different topic and/or longer periods of study.
4. The unusual grading system of the junior college used in

the study may have skewed the achievement level categories so that some low-achievers were called average-achievers and some average-achievers were called high-achievers. This skewing may have affected the results of the study within achievement levels.

5. The size of the sample population was smaller than originally anticipated. This resulted in some mean error scores being based on as few as one observation. The results obtained cannot be safely generalized to larger populations.

Implications

The present study contributes to the growing body of information about the effectiveness of the mathematics laboratory as a mode of instruction. Although none of the sequencing patterns was found to be more effective than the others, the significant difference between the two personality types does have some implications for further research. A study of personality interaction with a laboratory experience using all sixteen categories obtained from the Myers-Briggs Type Indicator might prove informative. Further research into the laboratory effectiveness within achievement levels also should be undertaken with greater rigor, than was possible in this study, on the definitions of high-achiever, average-achiever and low-achiever. As noted earlier the combination method would seem to be the best

approach at present for all achievement levels and all personality types.

Finally, it is hoped that this study will encourage other studies dealing with the mathematics laboratory as an adjunct to traditional modes of instruction at all levels of education.

APPENDIX A

PERFORMANCE OBJECTIVES AND PRETEST-POSTTEST

PERFORMANCE OBJECTIVES FOR RATIO AND SIMILARITY

1. The student should be able to write a ratio in the form (a:b) to compare the cardinalities of two sets.
2. The student should be able to write a ratio in the form (a:b) to compare the lengths of two line segments.
3. The student should be able to write an extended ratio in the form (a:b:c) to compare the cardinality of three sets.
4. The student should be able to write an extended ratio in the form (a:b:c) to compare the lengths of three line segments.
5. The student should be able to give two sets to illustrate a particular ratio.
6. The student should be able to give two line segments to illustrate a particular ratio.
7. The student should be able to give three sets to illustrate a particular extended ratio.
8. The student should be able to give three line segments to illustrate a particular extended ratio.
9. The student should be able to partition two sets and write the resulting equivalent ratio.
10. The student should be able to divide two line segments into a specified number of congruent pieces and write the resulting equivalent ratio.
11. The student should be able to translate verbal ratios into symbolic ratios of the form (a:b).

12. The student should be able to determine when two ratios are equivalent.
13. The student should be able to write (a:b) = (c:d) in the product form a x d = b x c.
14. The student should be able to supply the missing part of a ratio needed to make two ratios equal.
    Example: (a:__) = (c:d)
15. The student should be able to translate word problems involving ratios into equations and solve them.
16. The student should be able to determine whether or not two triangles are similar.
17. The student should be able to determine whether or not two rectangles are similar.
18. The student should be able to construct a triangle which is similar to a given triangle.
19. Given two similar figures, the student should be able to find the constant of proportionality.
20. The student should be able to determine whether polygons of more than four sides are similar or not.

PRETEST-POSTTEST

1. Write a ratio comparing the cardinality of the set $A = \{1,4,9,10\}$ to the cardinality of set $B = \{2,5,6,7,11\}$ in that order. Answer ________

2. Write a ratio comparing the length of line segment AB to line segment CD, in that order.

   A 1 3/4 B   C 1 D

   Answer ________

3. Write a ratio comparing the cardinality of $A = \{2,3,5,7\}$ $B = \{1,4,6,9,10\}$ and $C = \{1,3,4,5,8,9\}$, in that order.

   Answer ________

4. Write a ratio comparing the lengths of line segments AB, CD and EF, in that order.

   A 1 B   C 1 1/2 D   E 2 F

   Answer ________

5. Give an example of two sets whose cardinalities are in the ratio 7:5. Answer ________

6. Draw two line segments whose lengths are in the ratio 2:3. Answer

7. Give an example of three sets whose cardinalities are in the ratio 2:3:5.

   Answer A = ________ B = ________ C = ______

8. Draw three line segments whose lengths are in the ratio 1:2:3. Answer

9. Divide each of the following sets into equal subsets and express the resulting equivalent ratio.

```
       *   *                 *
    *    *    *           *   *
         *                      *
```

   Answer ________

10. Divide each of the line segments in half and express the resulting ratio.

    A 1 B   C 2 D

    Answer ________

11. Write a ratio to represent "for every two sweaters there are three skirts." Answer __________

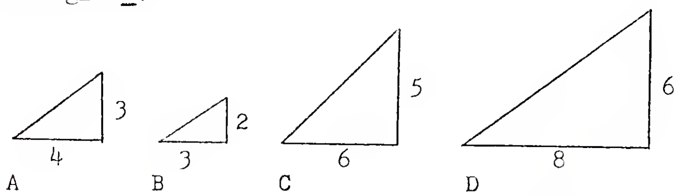
12. Which of the following ratios is equivalent to 3:4?
6:5 4:3 6:8 6:7 Answer __________

13. Express the statement (5:7) = (10:14) as a product.
Answer __________

14. Fill in the blank to make the statement (3:5) = (___:15) true. Answer __________

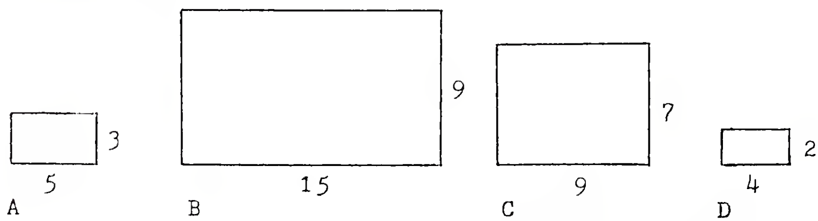
15. If a bicycle wheel makes 10 revolutions in going 33 feet how far does it travel in making 13 1/3 revolutions?
Answer __________

16. Which of the following triangles is similar to triangle A?



Answer __________

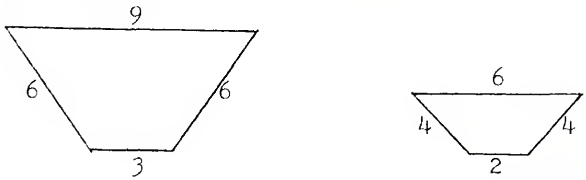
17. Which of the following rectangles is similar to rectangle A?



Answer __________

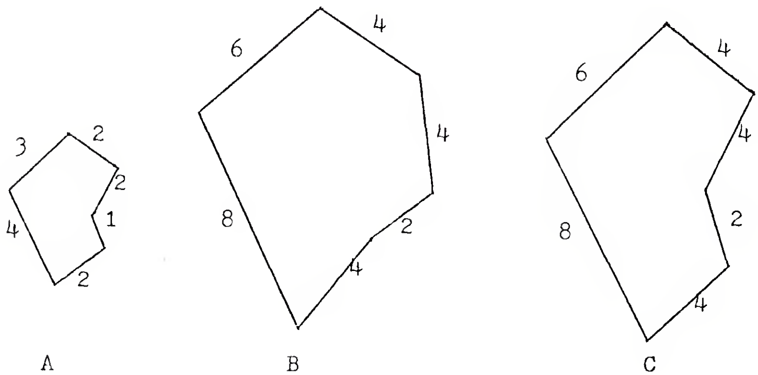
18. Construct a triangle similar to the one at the left with the indicated base line.

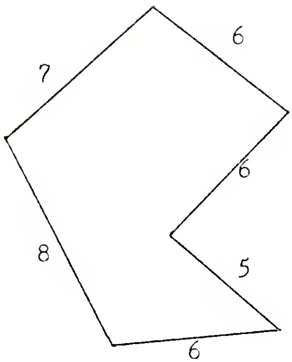
19. Given that the following two figures are similar, what is the constant of proportionality when comparing the left figure to the right figure?



Answer ____________

20. Determine which of the following figures is similar to figure A?





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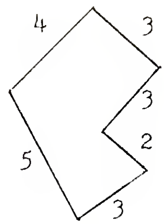


E

Answer ____________

APPENDIX B

EXPERIMENTS

EXPERIMENT 1

Problem: How do you use ratios to compare the numbers of objects in two sets?

Materials: Envelope containing 15 paper clips, 18 nails, 24 match sticks, 6 beans, 9 triangles, 24 squares

Procedures:

Example: Refer to Figure 1. The two sets are compared element to element. You can see that there are ______ more hexagons than squares.

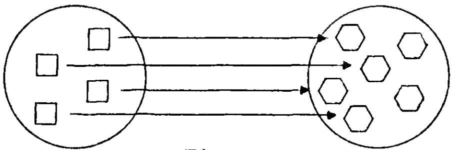


Fig. 1

Refer to Figure 2. The two sets are compared set to set. There are _____ squares and _____ hexagons. We can write the comparison in the form (4:6). We call this comparison a ratio.

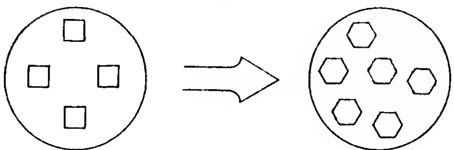


Fig. 2

Refer to Figure 3. Each set is separated into two equivalent subsets. They are compared subset to subset. The ratio (4:6) describes the comparison by sets. The ratio (2:___) describes the comparison by subsets.

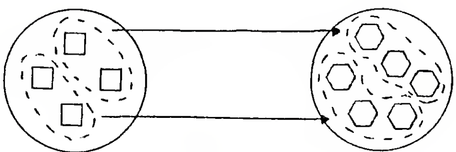
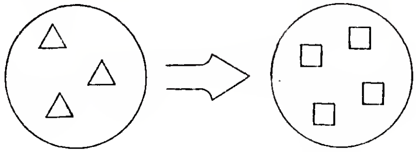


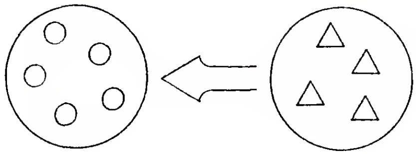
Fig. 3

1. Complete each ratio. Watch the direction of the arrow to see which set comes first.

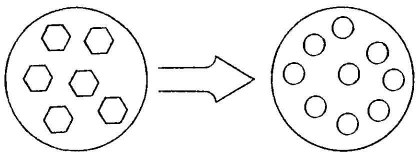
a) (3:____)



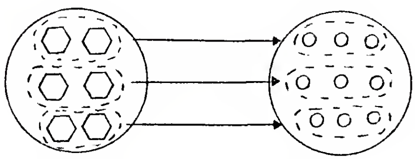
b) (____:5)



c) (____:____)



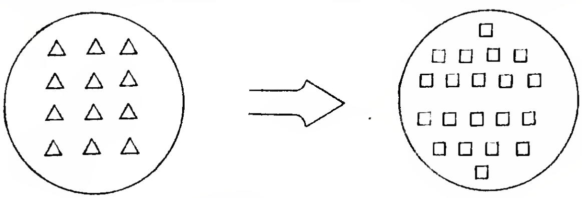
d) __________



2. In exercise 1 we compared (circle one)

a) the shapes of objects in two sets;
b) the sizes of objects in two sets;
c) the colors of objects in two sets;
d) the numbers of objects in two sets.

3. Write three ratios to compare the number of triangles with the number of squares in three different ways.



(12:____) (____:10) __________

The first ratio means there are twelve triangles for every ____________ squares.
The second ratio means there are ________ triangles for every ten squares.
This ratio can also be written (3:5), and it means ____
____.

4. Refer to the envelope of paper clips, nails, match sticks, and beans. Write a ratio for each of the following:

   a) The number of paper clips to the number of nails
   ____
   b) The number of match sticks to the number of beans
   ____
   c) The number of squares to the number of triangles
   ____

5. Here is how one group of students worked exercise 4.

   a) Tom wrote (15:18) to describe the ratio of the number of paper clips to the number of nails.
   Marcia wrote the ratio (5:6).
   Paul wrote (18:15).
   Which were right? ______________________________

   b) David wrote (6:24) to describe the ratio of the number of match sticks to the number of beans.
   Dana wrote the ratio (12:3) to describe the same ratio.
   Diane wrote (24:6).
   Which were right? ______________________________

   c) Three students wrote (3:8) to show the ratio of the number of squares to the number of triangles. Were they right? ______________

   Write two more ratios that show this comparison.
   ______________ and ______________

6. Write a ratio to compare the number of boys with the number of girls in your class. ______________

7. Write a ratio to represent each of the following:

   a) There are eighteen bicycles and thirty students.
   ____
   b) There are two bicycles for every three students.
   ____
   c) There are twice as many students as there are bicycles. __________

EXPERIMENT 2

Problem: How do you use ratios to compare lengths of objects?

Materials: 12-inch ruler, 1 red, 1 green, 1 yellow and 1 black stick, nail, clothespin, centimeter ruler

Procedures:

Example: Measure the red stick and the green stick in inches. The red stick is _____ inches long. The green stick is _____ inches long.

The ratio (____:6) can be used to compare the length of the red stick with that of the green stick. The ratio (1:____) could also be used. This means "one for every two." The red stick has a length of 1 inch for every _____ inches of the green stick.

1. Measure each object and record the length.

   a) red stick __________ in.
   b) green stick __________ in.
   c) yellow stick __________ in.
   d) black stick __________ in.
   e) nail __________ in.
   f) clothespin __________ in.

2. Write a ratio for each of the following:

   a) The length of the yellow stick to the length of the black stick. ____________
   b) The length of the nail to the length of the clothespin. ____________
   c) The length of a table to its width. ____________
   d) Your height (inches) to your weight (pounds). ______
   e) The length of the red stick to the length of the yellow stick. ____________
   f) The length of the green stick to the length of the black stick. ____________
   g) The length of the yellow stick to the length of the green stick. ____________
   h) The length of the classroom to the width of the classroom. ____________

3. Pat's desk is 12 ice-cream sticks long and 8 ice-cream sticks wide. Circle the ratios that can be used to compare its length with its width.

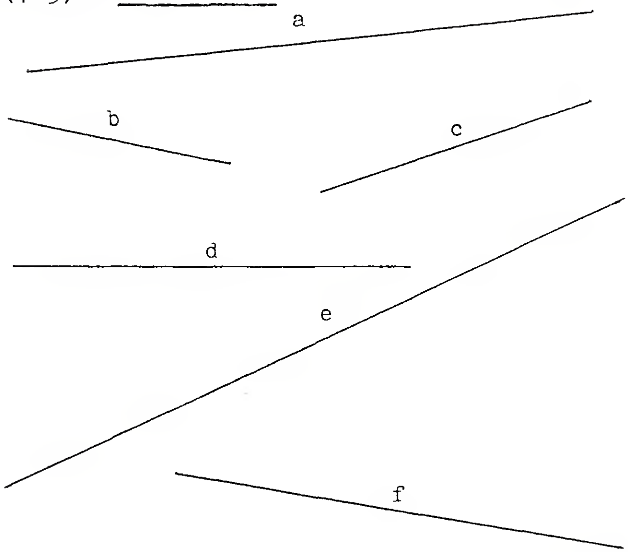
   a) (8:12) c) (6:4) e) (2:3)
   b) (12:8) d) (3:2) f) (4:1)

4. Write a ratio for each statement.

   a) A wall had 5 feet of width for every 2 feet of height. __________
   b) A dog was winning a tug-of-war with a boy, since there were 10 pounds of dog for every 8 pounds of boy. __________
   c) A very thin man weighs 130 pounds and is 74 inches tall. __________

5. Refer to the line segments below. Measure each segment using the centimeter ruler. Let each letter represent the measure of the line segment. Complete the following to indicate the ratios of these measures.

   a) (2:3) = (f:___)
   b) (5:2) = (___:___)
   c) (1:3) = (_________)
   d) (7:5) = __________



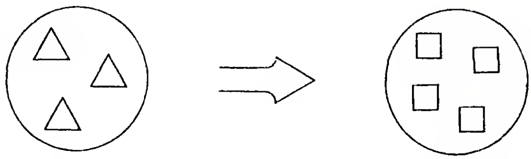
EXPERIMENT 3

Problem: How do you illustrate a given ratio?

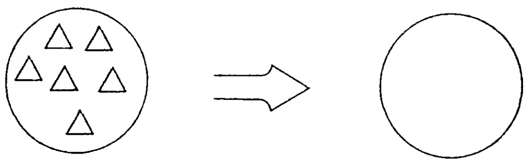
Materials: 10 red cubes, 20 blue cubes, 5 beans, 25 match sticks, and 12-inch ruler

Procedures:

1. Suppose we wish to illustrate a matching of a set of triangles with a set of squares that are in the ratio (3:4). To do this, we might draw the following:



Complete the following drawing to illustrate the same ratio:



2. A ratio for the number of tables to the number of students is (1:4). How many tables are there for 28 students? __________ Therefore (1:4) = (___:28).

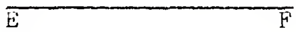
3. Place three red cubes and five blue cubes on the table. A ratio for the number of red cubes to the number of blue cubes is (___:___).

   Add six red cubes for a total of nine. How many blue cubes must you have so that there are three red cubes for every five blue cubes? ________ A ratio for the number of red cubes to the number of blue cubes is (9:____). We can say that (3:5) = (9:____).

4. Place four beans on the table. Place enough match sticks on the table so that the ratio of the number of beans to the number of match sticks is (1:6) = ________.

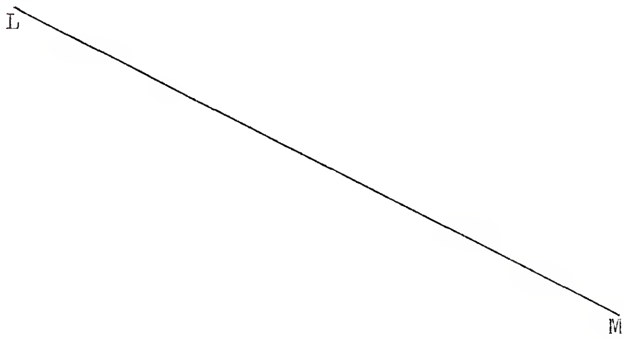
5. Draw line segments AB and CD so that a ratio for the length of segment AB to the length of segment CD is (2:3). If segment AB is four inches long, how long would CD be? ________________

6. Draw segment GH so that a ratio of the length of segment EF (below) to the length of segment GH is (1:2).



Segment EF must have one unit of length for every ______ units of length of segment GH.

7. Draw segment JK so that a ratio for the length of segment JK to the length of segment LM (below) is (2:3).



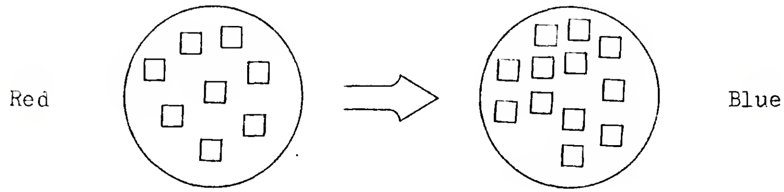
EXPERIMENT 4

Problem: How are ratios used to make comparisons?

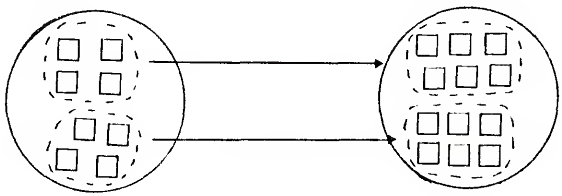
Materials: Envelope containing 8 red squares and 12 blue squares, egg beater

Procedures:

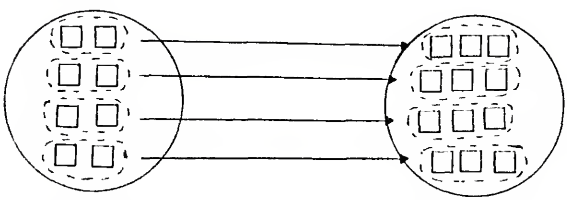
1. In the envelope there are eight red squares and ______ blue squares. The ratio of the number of red squares to the number of blue squares can be written as (8:12).



One student shows that there are four red squares for every six blue squares. He uses the ratio (4:6). A second student prefers to match two red squares with ______ blue squares. His ratio is (2:____). The set of red squares can be compared with the set of blue squares by any of these ratios: (8:12), (4:___), (____:3).



First student's comparison



Second student's comparison

The ratio (2:3) means ---

a) There are _____ red squares for every _____ blue squares.
b) There are two-thirds as many red squares as _____ squares.
c) For every _____ red squares there are _____ blue squares.

2. Turn the handle of the egg beater. While the handle makes one turn, the beater makes _____ turns. While the handle makes two turns, the beater makes _____ turns. We can use the ratio _____ to show that for every turn of the handle the beater makes _____ turns. The ratio (2:_____) could also be used.

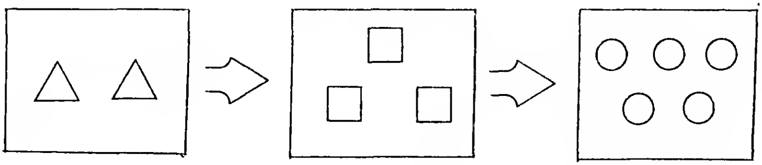
EXPERIMENT 5

Problem: How do you use extended ratios to compare the numbers of objects in three sets or to compare three measures?

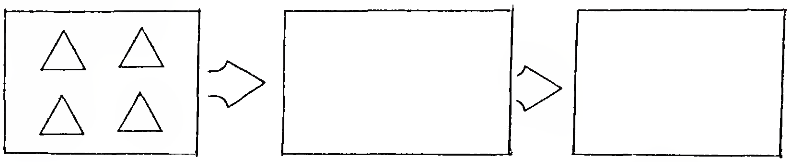
Materials: 3 green sticks, 3 blue sticks, 9 red cubes, 15 green cubes, 21 blue cubes, 12-inch ruler, and masking tape

Procedures:

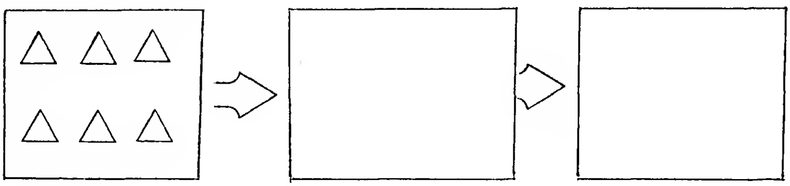
1. An extended ratio for the number of triangles to the number of squares to the number of circles, as pictured below, is (2:3:5).



Finish this drawing so that the number of triangles in the first rectangle is to the number of squares in the second rectangle is to the number of circles in the third rectangle as (2:3:5).



Finish this drawing so that it also shows the extended ratio (2:3:5).



In each of the drawings above, for every two triangles there are three squares and five circles. Make another drawing, different from the first three, in which the comparison of triangles with squares with circles is (2:3:5).

2. Complete the following extended ratio to show the comparison of the number of red, green and blue cubes: (3:____:____).

3. The three green sticks have lengths of 3 inches, 4 inches and 5 inches. The extended ratio that compares their lengths (from shortest to longest) is (3:____:____). The lengths of the three blue sticks are _____ inches, _____ inches and _____ inches. The extended ratio that compares their lengths (from shortest to longest) is (6:____:____).

4. Tape the green sticks together (as in Figure 4) to form a triangle. Do the same with the blue sticks. Do you notice anything about the two triangles?
____

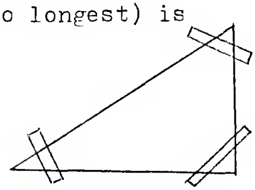


Figure 4

5. Dave plans to cut three red sticks so that their lengths can be compared by the extended ratio (3:4:5). If he cuts the shortest stick 9 inches long, how long should he make the other two? __________ and ______________

6. Sam Gravelcement makes concrete by mixing cement, sand, and gravel in the extended ratio (1:2:3). This means that for every measure of cement he uses _____ measures of sand and _____ measures of gravel. He has placed four buckets of cement in a cement mixer. How many buckets of sand must he add? ______ How many buckets of gravel? ______ Complete this proportionality so that it describes Sam's mixture: (1:2:3) = (4:____:____).

EXPERIMENT 6

Problem: How do you write equal ratios and extended ratios?

Materials: 18 triangles, 12 small squares, 16 large squares, 4 beans, and 24 match sticks

Procedure:

1. Place eight triangles and twelve small squares on the table. What is the ratio of the number of triangles to the number of squares?

2. Place the eight triangles and twelve squares in two equal piles on the table. (One pile should contain the same number of squares and the same number of triangles as the other pile.) For every four triangles there are ______ squares. This ratio can be written __________.

3. Next make four equal piles of the eight triangles and twelve squares. In each pile place two triangles and ______ squares. For every two triangles there are ______ squares. This ratio can be written __________.

4. You should have written three different ratios. Each ratio represents the same comparison of the number of triangles with the number of squares. Therefore these ratios are equal. That is, (8:__) = (__;6) = __________. An equation relating two ratios is called a proportion.

5. Place sixteen large squares on the table. Pretend they are sandwiches to be divided among four hungry students. A ratio for the number of sandwiches to the number of students is __________. Divide the sandwiches evenly among the four students. There are ______ sandwiches for each student. This ratio can be written (__:1). So, (16:4) = (__:1).

6. Place one bean and six match sticks on the table. Six match sticks are needed for each bean. The ratio of the number of match sticks to the number of beans must be (6:__). Place three more beans on the table with as many match sticks as are needed. Since we will need ______ match sticks for the four beans, (6:1) = (__:4).

7. Place eight triangles, twelve small squares and sixteen large squares in a pile on the table. The extended ratio that compares triangles with small squares with large squares is (__:12:__).

   Now make two equal piles. Place four triangles, ______ small squares, and eight large squares in each pile.

The extended ratio that compares the numbers of triangles, small squares, and large squares in each pile is (___:___:8).

Finally make four equal piles. Place ______ triangles, ______small squares, and ______ large squares in each pile. The extended ratio representing the comparison of triangles with small squares with large squares in each pile is ________________.

You should have written three different extended ratios. They all represent the same comparison, so they are equal to one another. That is, __________ = __________ = ______. An equation relating a pair of extended ratios is a proportionality.

8. Supply the numbers missing from each ratio (or extended ratio).

a) (7:3) = (___:9)
b) (___:2) = (20:4)
c) (1:12) = (5:___)
d) (1:2:3) = (___:___:21)
e) (___:15:17) = (16:___:34)
f) (___:1) = (10:2)
g) (8:24:26) = (___:12:___)

EXPERIMENT 7

Problem: When are two ratios equal?

Materials: 12-inch ruler with mm markings

Procedures:

1. Measure segments AB, CD, and EF in inches (to the nearest half inch) and in centimeters (to the nearest millimeter). Record these measurements in the following table. Predict the length of segment GH in centimeters.

RATIO

	Length in inches		Length in centimeters	
AB	(________	:	6.4	)
CD	(________	:	________	)
EF	(________	:	________	)
GH	(40	:	________	)

2. The ratio (2.5:6.4) means that for every 2.5 inches of length there are 6.4 centimeters of length.

   If the terms of (2.5:6.4) are doubled, we get (____:____).
   If the terms of (2.5:6.4) are tripled, we get (____:____).

3. Wilt Stilton is mixing concrete for his basketball area. He is using the extended ratio (1:2:3) to represent his mixture of cement, sand and gravel. This means that he mixes one part cement _____ parts sand, and _____ parts gravel.

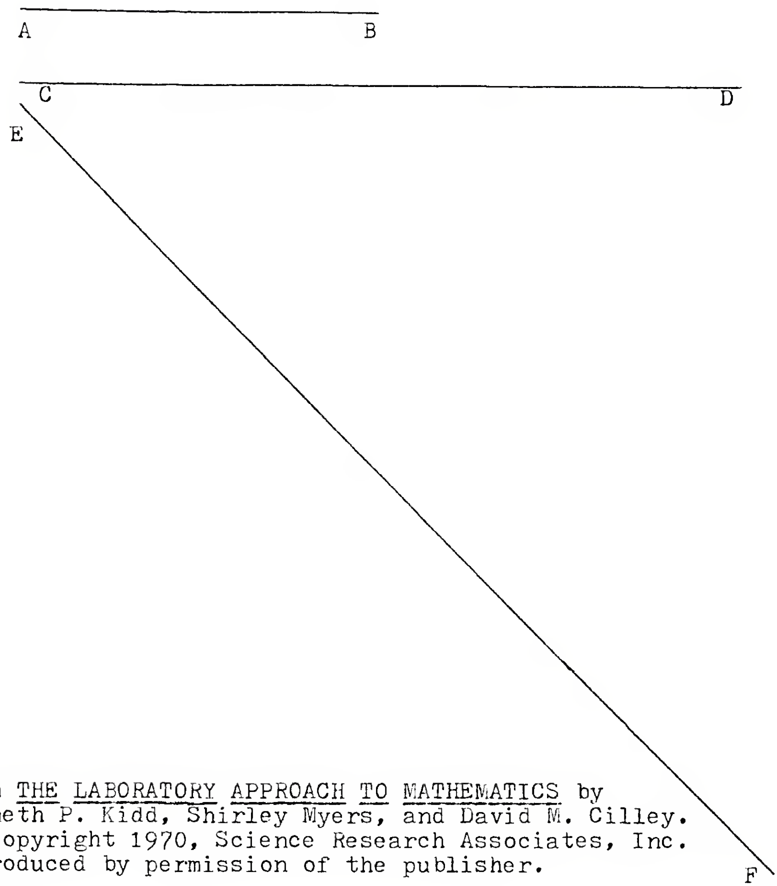
   Wilt plans to put four shovelfuls of cement in the mixer. How many shovelfuls of sand and of gravel must he use?
   (1:2:3) = (4:____:____)
   (1:2:3) means that for each shovelful of cement he uses, he must use _____ shovelfuls of sand and _____ of gravel. How can the missing numbers in the extended ratio above be found? ______________________________

4. There are six pizzas for four people, so there are three pizzas for two people, or 1 1/2 pizzas for each person. Complete the following ratios. Each can be used to compare the number of pizzas with the number of people.
   (6:____)   (____:____)   (1 1/2:____)
   The three ratios are equal. The second ratio can be obtained by multiplying each term of (6:4) by 1/2. If we multiply each term of (6:4) by 1/4, we get (1 1/2:____).

<u>Definition</u>: If k is any positive number, (a:b) = (ka:kb) and (a:b:c) = (ka:kb:kc).

5. Fill in the missing numbers.

   a) (3:8) = (9:____)
   b) (5:3) = (____:1)
   c) (20:5) = (____:1)
   d) (3:4:5) = (12:____:____)
   e) (8:8) = (1:____)



EXPERIMENT 8

Problem: Do equal ratios have any special properties?

Materials: None

Procedures:

1. Fill in the missing numbers to make pairs of equal ratios:

   a) (8:6) = (4:___)
   b) ($4\frac{1}{2}$:2) = (___:4)
   c) (6:4) = (___:1)
   d) (2.54:1) = (___:10)
   e) (9:5) = (___:2)
   f) (2:8) = (___:1)

2. Suppose we start with any ratio (a:b), where a,b $\neq$ 0. How can we write a different ratio that is equal to (a:b)?

   ____

3. A proportion is a statement that 2 ratios are equal. A proportion has 4 terms. The outside ones are called the extremes. The inside ones are the means. In (9:2) = (18:4), the 9 and the 4 are the extremes and the 2 and the 18 are the means.

   For (9:2) = (18:4), find the product of the extremes. ___
   For (9:2) = (18:4), find the product of the means. ___
   How do the product of the extremes and the product of the means compare? ____________________

4. Find the product of the extremes and the product of the means for each of the following proportions.

Proportion	Product of the Extremes	Product of the Means
(30:5) = (6:1)		
(6:4) = ($1\frac{1}{2}$:1)		
($4\frac{1}{2}$:2) = (9:4)		
(5:12.7) = (10:25.4)		

   How do the two products compare in each case? __________

   ____

> Product rule: In a true proportion the product of the extremes equals the product of the means.

5. Follow the rule to write each proportion as an equation involving products.

a)	(2:3) = (8:12)	2 x 12 = 3 x ___
b)	(4:2) = (2:1)	4 x ___ = ___ x 2
c)	(2:3) = (1:1$\frac{1}{2}$)	2 x ___ = ___ x ___
d)	(3:8) = (6:16)	___ x ___ = ___ x ___
e)	(4:10) = (10:25)	______________________
f)	(54:10) = (__:1)	______________________
g)	(32:__) = (64:4)	______________________

6. Solve each proportion for ___.

a) (4:___) = (2:6)
b) (4:12) = (10:___)
c) (120:2$\frac{1}{2}$) = (___:1)
d) (3:5) = (___:2$\frac{1}{2}$)
e) (___:24) = (3:4)
f) (50:7.39) = (____:23.42)

EXPERIMENT 9

Problem: How do you use a measuring wheel?

Materials: 50-foot tape, measuring wheel and two stakes.

Procedure:

1. Roll the measuring wheel through two turns. Measure the distance the wheel traveled in making two full turns. Write a ratio for the number of turns of the wheel to the number of feet it traveled. ____________

2. Now roll the wheel in a straight line for twelve turns. The ratio of the number of turns to the number of feet traveled is (12:___). Find a number to replace ___. Your ratio in the first problem was __________. So __________ = (12:___). What must replace ___?

3. Use the tape to measure the distance for twelve turns of the wheel. ______feet. How does this compare with your answer in exercise 2? ____________

4. How many turns must you roll the wheel to measure 100 feet? (2:5) = (___:100) In each ratio the first number is the number of __________. The second number is the number of __________. What is your replacement for ___?

5. Measure a distance of 100 feet. Roll your wheel. How many turns did it take? ________ How does your answer compare with your replacement for ___ in exercise 4?

   ____

6. Using your measuring wheel, place two stakes in the ground 50 yards apart. These are the starting and finishing positions for the 50-yard dash. Measure the distance between the stakes with the tape. __________ How does this answer compare with the answer you got in exercise 5? ____________

7. Roll the wheel for twenty-five turns. What should the distance be? __________ Check the distance with the tape.

8. Measure a distance (say, 75 feet) along some curved path around the school. Ask another member of your team to measure the same path. Compare your results. ________

   ____

9. How many turns will the wheel make in going a mile?

____

EXPERIMENT 10

Problem: How can you use ratios to measure speed?

Materials: 50-foot tape, measuring wheel, stop watch

Procedures:

1. Measure the distance you can walk in ten seconds. Write a ratio for the number of feet walked to the number of seconds of walking. (____:10)

2. If you continued walking at this rate, you would be going ________ feet every 10 seconds. How far would you go in 60 seconds? (____:10) = (____:60) (Find a replacement for the ____.) In these ratios the first term is the number of feet and the second term is the number of ________________.

3. Walk in a straight line at normal speed for 60 seconds. Use the measuring wheel to measure the distance you walked. ____________________

4. Wally Walker walked 50 feet in 10 seconds. His ratio of the number of feet traveled to the number of seconds of traveling is (50:10) or (5:1). We say that Wally's walking speed is 5 feet per second, or 5fps. How many seconds would it take Wally to walk one mile (5280 feet) at this speed? This is the same as _______ minutes and __________ seconds.

5. Ratios are frequently used to show speed. The second term of the ratio is usually 1.

   When comparing the number of feet traveled with the number of seconds of traveling ---
   a) the ratio (4:1) means 4 feet per second;
   b) the ratio (6:1) means _________________; and
   c) the ratio ________ means 9 feet per second.

6. Roger Bannister was the first person to run one mile in less than four minutes. If he could have run for one hour at the rate of four minutes per mile, how many miles could he have run? His average speed was _____ miles per hour (mph). One mile is ________ feet. Four minutes is _______ seconds. Therefore the ratio of the number of feet traveled to the number of seconds of traveling is (5280:240). (5280:240) = (___:1). What is the correct replacement for ___? Therefore we know that _____ fps is the same as 15 mph.

EXPERIMENT 11

Problem: What is a ratio compass and how do you use it?

Materials: Ratio compass, centimeter ruler, worksheet 11

Procedures:

1. Place the bolt in the hole marked (2:3) on the ratio compass and tighten the nut on the bolt. The compass now has two short arms and two long arms. Open the compass until the ends of the short arms are 10 centimeters apart. Measure the distance between the ends of the longer arms. ________cm. Draw line segments AB and CD and complete the following: (length of segment AB: length of segment CD) = (2:3) = (10:___).

2. Complete the following chart for the (2:3) setting.

Length of segment AB (cm.)	10			18	
Length of segment CD (cm.)	15	18	21		36

   For any opening of the compass, (length of segment AB: length of segment CD) = (2:3). Name the other ratios that can be set on the compass. If you have time, make a chart for each ratio like the one for (2:3).

3. Worksheet 11 shows five line segments, each named AB. For each, use the compass to construct another line segment (name it CD) such that (length of segment AB: length of segment CD) = (2:3). In each case use a proportion to find the length of CD.

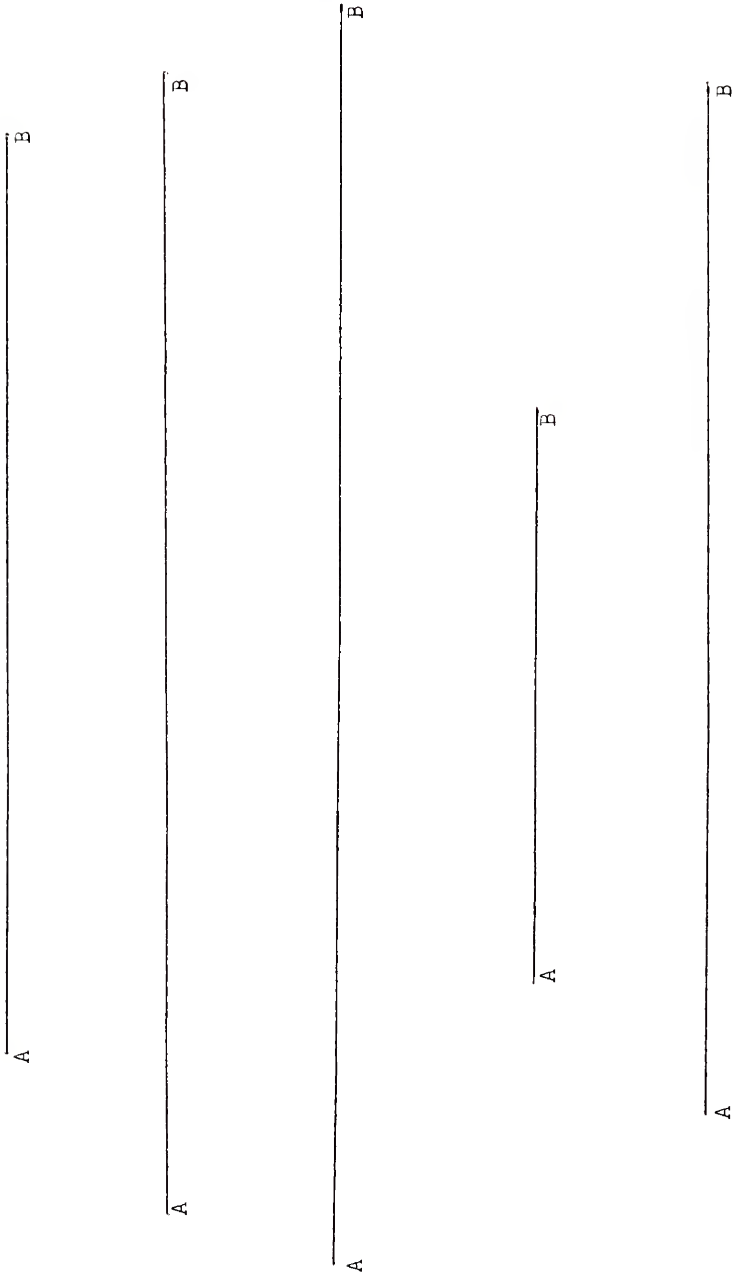
   a) Length of segment AB is _______ cm.
   (___:___) = (2:3)
   Length of segment CD is _______ cm. (From above)
   Check by measuring CD
   b) Length of segment AB is _______ cm.
   (___:___) = (2:3)
   Length of segment CD is _______ cm. (From above)
   Check by measuring CD
   c) Length of segment AB is _______ cm.
   (___:___) = (2:3)
   Length of segment CD is _______ cm.
   Check by measuring CD
   d) Length of segment AB is _______ cm.
   _________ = _________
   Length of segment CD is _______ cm.
   e) Length of segment AB is _______ cm.
   ____
   Length of segment CD is _______ cm.

4. On the poster board is a triangle KLM such that side KL is 18cm. long, side LM is 24 cm. long, and side MK is 30 cm. long. There is a 90^{o} angle between sides KL and LM. With the ratio compass set at (2:3), adjust the larger opening to match side LM. Draw a line segment equal in length to the distance between the points at the smaller opening of the compass. Label it EF. Now adjust the larger opening of the ratio compass to side KL. Draw a line segment equal in length to the distance between the points at the smaller opening of the compass. Draw this segment at point E so that if forms a 90^{o} angle with segment EF. Label it EG. Adjust the larger opening of the ratio compass to side MK. Compare the smaller opening with the distance between points F and G.

5. Use proportions and the lengths of the sides of the larger triangle to predict the lengths of the sides of the smaller triangle.

   Measure the sides of the smaller triangle. How do these lengths compare with the results you got using proportions?

WORKSHEET 11



EXPERIMENT 12

Problem: How can ratios help to predict events?

Materials: Thumbtack, solid with 4 colored faces (each face an equilateral triangle), and a pair of dice

Procedures:

1. When you toss a thumbtack, it will land in either of two ways. Toss a thumbtack twenty times. How many times did it land in position A (point up)? in position B? Guess the number of times it will land each way if you toss it forty times. Toss the tack twenty more times and total your results for forty tosses.

	A	B
20 tosses		
Guess for 40 tosses		
Results for 40 tosses		

2. The object with the four colored faces can land with any of the faces down.

Face that is down	red	yellow	green	blue
Number of tosses				

Throw the solid fifty times. Chart the number of times it falls with each color down. For a single throw, which color is most likely to be down? ________________

What is your best guess for the number of times green would be down in 100 throws? ______________

3. Toss a pair of dice 36 times and complete the following table.

	Sum of numbers on upper faces of the two dice										
	2	3	4	5	6	7	8	9	10	11	12
Number of times the given sum was obtained in 36 rolls											
Guess of the number of times the given sum would be obtained in 76 rolls											
Number of times the given sum was obtained in 76 rolls											

How did you make your guesses and how good were they? ______________________

____

From THE LABORATORY APPROACH TO MATHEMATICS by Kenneth P. Kidd, Shirley Myers, and David M. Cilley.

EXPERIMENT 13

Problem: How can you use a bicycle wheel to measure distance?

Materials: 50-foot tape, bicycle wheel mounted on a fork, chalk, and string

Procedures:

1. Fix a mark on the ground. Starting at the mark, roll the wheel in a straight line. Measure (to the nearest tenth of a foot, 1 inch = .08 foot) the distance the wheel traveled in making ten turns. __________feet What is the ratio of the number of turns of the wheel to the number of feet traveled? __________________

2. Fix two marks (A and B) on the ground about 70 paces apart. Use the bicycle wheel and a proportion to find the distance from A to B. (Remember: roll the wheel in a straight line.) Do this twice to check for accuracy. Then set up the following proportion:

   (10:_______) = (__________:__________)

distance traveled in making ten turns	number of turns from A to B

   Find the correct replacement for the last blank. What does this represent?

   Use the tape to measure the distance from A to B. _____________feet How do your answers compare? _______ ____ (If your answers differ by more than $\frac{1}{2}$ foot, try again.)

3. Now mark off 100 paces with two stakes. Measure the distance, using the bicycle wheel and complete the following proportion.

   (10:_______) = (__________:__________)

distance traveled in making ten turns	number of turns between the marks

4. Take the piece of string which is 5 feet long and use it to draw a circle with a radius of 5 feet. Make sure the string is taut and straight at all times. Measure the distance around this circle, using your bicycle wheel. Distance: ___________________

5. Measure the distance along some path. Make the measurement twice. Compare your results with those of your classmates.

EXPERIMENT 14

Problem: How do you measure distances on a globe?

Materials: Globe of the earth, string

Procedures:

1. Locate the equator on the globe. The earth's equator is 25,000 miles long and contains 360 arc degrees. The curved lines that come together at the poles are called meridians. Find the meridian through Greenwich, England (near London). Locate the point where this meridian crosses the equator. This will be our zero point. From here we will measure distances along the equator. The equator is marked off (in each direction) in arc degrees. Find where the equator passes through the Isle of Celebes (in the East Indies). It is ______ arc degrees from our zero point. Therefore (__________:__________) = (360:25,000). So, the distance in miles from our zero point to the Isle of Celebes is ________________.

2. How far is it from Chicago, Illinois, to Cairo, Egypt? The shortest path can be found by stretching a string from Chicago to Cairo on the globe. Can you think of a way of finding the number of arc degrees in the path? This path contains ______ arc degrees. Complete this proportion. Then solve it to find the distance from Chicago to Cairo. (360:25,000) = (______:__________)

3. Jacksonville, Florida, and Shanghai, China, are at about the same latitude (31° N). Find the shortest path from Jacksonville to Shanghai. If you flew along this path, would you go east or west of St. Louis, Missouri? ______ Would your flight take you north of the Arctic Circle? ________ What would be the length of your flight in arc degrees? ___________ in miles? _____________ How far would you fly if you traveled westward along the 31° N latitude line from Jacksonville to Shanghai? ___________

4. How far is it from Buenos Aires, Argentina to Paris, France? _____________ arc degrees; _____________ miles

From THE LABORATORY APPROACH TO MATHEMATICS by Kenneth P. Kidd, Shirley Myers, and David M. Cilley.

EXPERIMENT 15

Problem: What do ratios have to do with the enlargement of figures?

Materials: Cardboard polygons (ABCD and PQRS), centimeter stick, drawing of a dog, and a sheet of tracing paper

Procedures:

1. Do the cardboard polygons have the same shape? Compare each angle of polygon ABCD with each angle of polygon PQRS. Draw lines to show the pairs of angles that have the same measure.

$\angle$ DAB	$\angle$ SPQ
$\angle$ CBA	$\angle$ PQR
$\angle$ DCB	$\angle$ SRQ
$\angle$ CDA	$\angle$ PSR

2. Draw lines to show the pairs of corresponding sides and diagonals. Measure each side and each diagonal, and complete the following table.

First polygon	Second polygon
AB	PQ
BC	QR
CD	RS
DA	SP
AC	PR
BD	QS

Segment	Length
AB	
BC	
CD	
DA	
AC	
BD	
PQ	
QR	
RS	
SP	
PR	
QS	

3. Using the measurements from the table above, complete the following ratios.

(length of AB : length of SP) = (18:12) = (3:2)
(length of BC : length of PQ) = ______ = ______

(length of CD : length of QR) = ______ = ______
(length of DA : length of RS) = ______ = ______
(length of AC : length of SQ) = ______ = ______
(length of BD : length of PR) = ______ = ______

4. There are two drawings of a dog. One is an enlargement of the other. Do the dogs in the drawings have the same shape? ______ Measure each of the following segments.

Segment	Length	Segment	Length
BC	10.0 cm.	EF	______ cm.
AC	______ cm.	DF	______ cm.
AB	______ cm.	DE	______ cm.

Complete the following ratios:

(length of BC : length of EF) = (10:____) = (1:____)
(length of AC : length of DF) = ______ = ______
(length of AB : length of DE) = ______ = ______

5. Put a sheet of tracing paper over the larger drawing and copy points D, E, and F. Cut out triangle DEF.

Does $\angle$ E have the same measure as $\angle$ B? ______

Does $\angle$ D have the same measure as $\angle$ A? ______

Does $\angle$ F have the same measure as $\angle$ C? ______

6. If two figures have the same shape, it appears that the following statements are true:
a) Corresponding angles have ______________________.
b) Corresponding sides and diagonals are ____________
______________________________.

EXPERIMENT 16

<u>Problem</u>: If two figures have the same shape, how do the lengths of their sides compare?

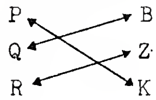
<u>Materials</u>: Envelope containing 2 cardboard triangles and 2 cardboard quadrilaterals, 12-inch ruler-cm scale, protractor

<u>Procedures</u>:

1. Refer to the triangles in the envelope. What do you notice about their shapes?

   The triangles should be matched in the following way:

   <u>Vertices</u>

   Draw lines to match the sides:

   <u>Sides</u>

PQ	BZ
QR	ZK
RP	BK

   Measure the sides of the triangles:

   PQ __________ cm. BZ __________ cm.
   QR __________ cm. ZK __________ cm.
   RP __________ cm. BK __________ cm.

   Write the ratios of the pairs of matching sides:

   (length of side PQ : length of side BK) = ____________
   (length of side QR : length of side BZ) = ____________
   (length of side RP : length of side ZK) = ____________

   Are the ratios you just wrote equal to each other? ______

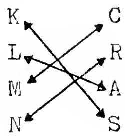
   If they are all equal, what is the simplest ratio equal to all three? ____________

2. Refer to exercise 1. Using the lengths of sides PQ, QR, and RP and the ratio you found, find the lengths of sides BK, BZ and ZK. Are these the same measures that you wrote down in exercise 1? If not check your work.

3. Refer to the two triangles. What do you notice about the angles at corresponding vertices of the two triangles?

> **Definition:** If two triangles are the same shape, the following are true:
> a) Matching pairs of angles are the same size.
> b) The ratios of the measures of the matching sides are equal.

4. Refer to the cardboard figures KLMN and CRSA. The matching vertices are:



The lengths of the sides of the two figures are:

KL	________	cm.	CR	________	cm.
LM	________	cm.	RS	________	cm.
MN	________	cm.	SA	________	cm.
NK	________	cm.	AC	________	cm.

Connect the names of the matching sides.

KL	CR
LM	RS
MN	SA
NK	AC

Are the matching sides the same length? ________
Are the matching angles the same size? ________ Which of these gives the ratio of the pairs of matching sides?

(1:2) (4:13) (1:3) (5:13)

> **Definition:** If two four-sided figures are the same shape, the following are true:
> a) Matching pairs of angles are the same size.
> b) The ratios of the measures of the matching sides are equal.

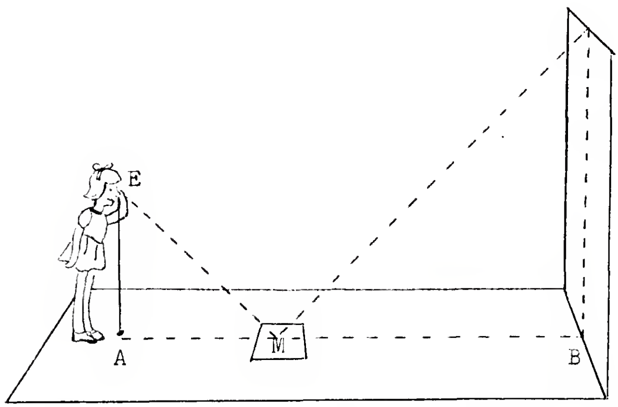
EXPERIMENT 17

Problem: How do you use a mirror to find the height of an object?

Materials: Mirror, marble for leveling the mirror, string and weight, felt-tip pen, yardstick

Procedures:

1. Select an object whose height you would like to find. (The area around its base must be level.) Mark a point in the middle of the mirror with the felt-tip pen. Label the point M. Lay the mirror flat of the floor and level it.



2. Stand where you can do both of the following (see the figure).

    a) Lean slightly forward and hold a plumb line to your eye (E) so that it touches the floor at a point (A).
    b) See the top of the object at M on the mirror.

3. Have a partner mark point A and point B at the base of the object you are measuring.

Measure segment EA ________________ 1 inch = .08 ft.
Measure segment MA ________________
Measure segment MB ________________

4. How are triangle EAM and triangle CBM related? Use a proportion and the results of exercise 3 to find the length of BC.

5. Repeat the experiment, placing the mirror a different distance from the foot of the object. How do your two results compare? ______________________________

EXPERIMENT 18

Purpose: To introduce the student to the idea of similarity for triangles and quadrilaterals.

Materials: Envelope containing red, blue and white triangles, and red, blue and white quadrilaterals.

Procedures:

1. The red cardboard triangles are alike in some way other than color. The blue triangles are not like the red triangles in this way. In what way are the red triangles alike? ______________________________
Which of the white cardboard triangles are like the red triangles in this way? ______________________

2. The red quadrilaterals are alike in some way other than color. The blue quadrilaterals are not like the red quadrilaterals in this way. In what way are the red quadrilaterals alike? ______________________________ Which of the white quadrilaterals are like the red quadrilaterals? ______________________________

3. From this we see that two triangles are alike if they have the same ________________. Likewise, two quadrilaterals are alike if they have the same ______________________.

4. Two triangles which have the same shape are said to be similar. Not all triangles are ____________.

5. Two quadrilaterals which have the same shape are said to be similar. Not all quadrilaterals are ____________.

EXPERIMENT 19

Purpose: To illustrate the one-to-one correspondence between vertices and sides of similar triangles and quadrilaterals.

Materials: Red triangle, white triangle, yellow quadrilateral and white quadrilateral

Procedures:

1. The red triangle ABC has the same shape as the white triangle DEF. Establish a one-to-one correspondence between the corresponding vertices of the two triangles and the corresponding sides. Draw lines connecting the corresponding parts.

Vertices		Sides	
A	D	AB	ED
B	E	BC	FD
C	F	AC	EF

2. The yellow quadrilateral LNKM has the same shape as the white quadrilateral RPTS. Establish a one-to-one correspondence between the corresponding vertices and sides of the two quadrilaterals. Draw lines connecting the corresponding parts.

Vertices		Sides	
L	S	LN	ST
N	R	NK	RP
K	P	MK	PT
M	T	LM	SR

EXPERIMENT 20

Purpose: To show how one triangle may be used to construct a similar triangle.

Materials: Yellow cardboard triangle, ruler

Procedures:

1. The yellow cardboard triangle ABC has a side AB which is 15 inches long. Use only the yellow cardboard triangle and a pencil to draw triangle A'B'C' which is similar to ABC, on line A'B' which is 5 inches long.

   Measure B'C' ______________
   How long do you think BC is? (Do not measure.) ________
   Measure BC ______________

   Measure A'C' ______________
   How long do you think AC is? (Do not measure.) ________
   Measure AC ______________

2. Write a ratio comparing BC to B'C' ______________
   Write a ratio comparing AC to A'C' ______________
   What is the relationship between these two ratios? _____
   ____
   This ratio could also be expressed as (____:1).

3. To change triangle ABC into triangle A'B'C', it would be necessary to multiply the length of each side by _______. This number is called the constant of proportionality.

____

A' B'

EXPERIMENT 21

Purpose: To show that two angles and one side are sufficient to construct a similar triangle.

Materials: Yellow triangle that has been cut apart, ruler.

Procedures:

If two triangles are similar, then the ratio of the three pairs of corresponding sides is equal. The yellow triangle has been cut apart. Leave the top of the yellow triangle folded under. Make a small triangle that is similar to the larger triangle. You may use any ratio of corresponding sides that you care to use. With this information compute the length of AC and BC of the large yellow triangle without actually measuring.

Length of AC is ________________.
Length of AC using a ruler is ________________.

Length of BC is ________________.
Length of BC using a ruler is ________________.

Choose a different ratio and repeat the above procedure.

Length of AC is ________________.
Length of AC using a ruler is ________________.

Length of BC is ________________.
Length of BC using a ruler is ________________.

EXPERIMENT 22

Purpose: To show that corresponding angles in similar figures have the same measure.

Materials: Two similar pictures of Snoopy with points KLRNB marked on one and K'L'R'N'B' marked on the other, ruler, and protractor

Procedures:

Here are two similar pictures of Snoopy. You want to (1) find the ratio of corresponding line segments, and (2) compare the size of the corresponding angles.

Measure to the nearest 1/16 of an inch

KL ________ K'L' ________

KR ________ K'R' ________

KN ________ K'N' ________

KB ________ K'B' ________

Ratios of measures of corresponding distances

(1: ______)

(1: ______)

(1: ______)

(1: ______)

Draw lines KL, KR, KN and KB. Also draw line K'L', K'R', K'N', and K'B'. Measure angle NKL and N'K'L'. What is the relationship between these two angles? ____________________ ____________________ Compare the size of the other angles. What do you observe about each pair of angles? __________ ____

EXPERIMENT 23

Problem: To stress the importance of establishing all three conditions needed to show similarity between two polygons if the polygons are of more than three sides.

Materials: Ruler

Procedures:

Look at the two hexagons on the next page.

1. If possible, draw lines showing one-to-one correspondences between corresponding vertices and corresponding sides.

VERTICES		SIDES	
A	B'	AB	E'F'
B	E'	BC	A'B'
C	D'	CD	D'E'
D	A'	DE	F'A'
E	C'	EF	B'C'
F	F'	FA	C'D'

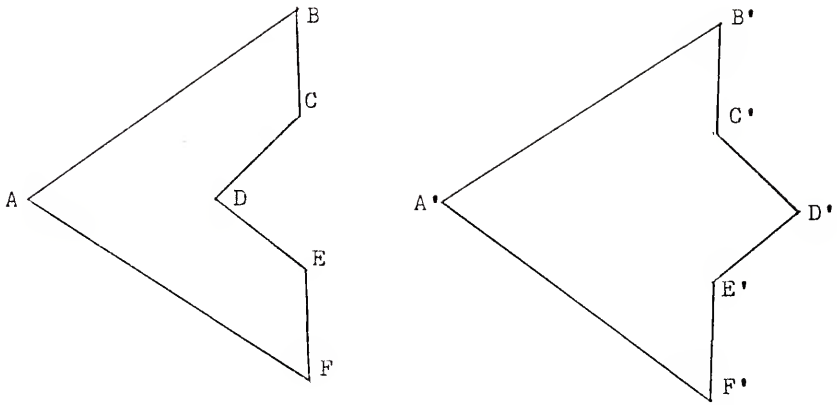
2. Are the corresponding sides proportional? ________ If so, what is the ratio of proportionality? __________

3. What can be said about the measure of corresponding angles?

ABC	_____	A'B'C'	DEF	_____	D'E'F'
BCD	_____	B'C'D'	EFA	_____	E'F'A'
CDE	_____	C'D'E'	FAB	_____	F'A'B'

4. Are the hexagons similar? __________________

5. What three things must be true for figures of more than three sides to be similar? ____________________________
____



B
C
A
D
E
F
B'
C'
A'
D'
E'
F'

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BIOGRAPHICAL SKETCH

Joan Marie Golliday was born July 28, 1942, in Woodbine, Maryland. At a few months of age, she moved with her parents to Sykesville, Maryland, where she attended public school. She graduated from Sykesville High School in June, 1960, and in September of that same year, enrolled at Florida Southern College in Lakeland, Florida. While there, she pursued an undergraduate program in mathematics and received the degree of Bachelor of Science, summa cum laude, in May of 1964. For the next two years, she held the position of Graduate Teaching Assistant in the mathematics department of the University of Florida. In August, 1966, she received the degree of Master of Arts in mathematics. From September, 1966, until June, 1968, she taught mathematics at Santa Fe Community College while continuing as a part-time graduate student in the College of Education at the University of Florida. In June of 1968, she was awarded a one-year Graduate School Fellowship which enabled her to attain the degree of Specialist in Education in August 1969. Since September of 1969, she has been employed at Santa Fe Community College while completing the requirements for the degree of Doctor of Philosophy with specialization in mathematics education.

Joan Marie Golliday is a member of Phi Kappa Phi, Pi Lambda Theta, Kappa Delta Pi, the Florida Council of Teachers of Mathematics, the Florida Junior College Council of Teachers of Mathematics, and the National Council of Teachers of Mathematics.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Kenneth P. Kidd

Dr. Kenneth P. Kidd, Chairman
Professor of Education

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Elroy J. Bolduc

Dr. Elroy J. Bolduc, Cochairman
Associate Professor of Education

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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Dr. Vynce A. Hines
Professor of Education

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Charles W Nelson

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Associate Professor of
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This dissertation was submitted to the Dean of the College of Education and to the Graduate Council, and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

August, 1974

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